Class – IX	Topic – Rational and Irrational number

1. Without actual division find which of the following rationals are terminating decimal:

(i) $\frac{9}{25}$

(ii) $\frac{7}{12}$

(iii) $\frac{121}{125}$

(iv) $\frac{37}{78}$

Solution:

(i) In $\frac{9}{25}$, the prime factors of denominator 25 are 5, 5. Thus, it is terminating decimal.

(ii) In $\frac{7}{12}$, the prime factors of denominator 12 are 2, 2 and 3. Thus, it is not terminating decimal.

(iii) In $\frac{121}{125}$, the prime factors of denominator 125 are 5, 5 and 5. Thus it is terminating decimal.

(iv) In $\frac{37}{78}$, the prime factors of denominator 78 are 2, 3 and 13. Thus, it is not terminating decimal.

2. Express each of the following as a rational number in the form of p/q where q ≠ 0.

(i) $0.\overline{6}$

(ii) $0.\overline{43}$

(iii) $0.\overline{227}$

(iv) $0.\overline{2104}$

Solution:

(i) Let x = $0.\overline{6}$ = 0.6666 ...(i)

Multiplying both sides of eqn. (i) by 10, we get

10x = 6.6666 ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

106.6666

x= 0.6666

$9x = 6 \Longrightarrow x = \frac{6}{9} = \frac{2}{3}$ Hence, required fraction $= \frac{2}{3}$

(ii) Let x $=0.\overline{43}$ = 0.43434343 ...(i)

Multiplying both sides of eqn. (i) by 100, we get

100x = 43.434343 ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

100 x=43.434343

x=0.434343

$99x = 43 \Longrightarrow 99x = 43$

$\Longrightarrow x = \frac{43}{99}$

Hence, required fraction $\frac{p}{q} = \frac{43}{99}$

(iii) Let x = $0.\overline{227}$= 0.2272727... ... (i)

Multiplying both sides of eqn. (i) by 10, we get

10x = 2.272727... ... (ii)

Multiplying both sides of eqn. (ii) by 100, we get

1000x = 227.272727... ... (iii)

Subtracting eqn. (ii) from (iii), we get

1000 x= 227.272727...

10x= 2.272727...

$990x = 225 \Longrightarrow 990x = 225 \Longrightarrow x = \frac{225}{990} = \frac{5}{22}$

Hence, required fraction $\frac{p}{q} = \frac{5}{22}$

(iv) Let x = $0.\overline{2104}$= 0.2104104104... ... (i)

Multiplying both sides of eqn. (i) by 10, we get

10x = 2.104104104... ... (ii)

Multiplying both sides of eqn. (ii) by 1000, we get

10000x = 2104.104104104... ... (iii)

Subtracting eqn. (ii) from (iii), we get

10000x= 2104.104104104

$10x = 2.104104104$

$9990x = 2102$

$\Longrightarrow 9990x = 2102$

$\Longrightarrow x = \frac{2012}{9990} = \frac{1051}{4995}$

$\text{Hence, required fraction} = \frac{1051}{4995}$

3. Insert one rational number between:

(i) $\frac{3}{5}$ and $\frac{7}{9}$

(ii) 8 and 8.04

Solution:

(i) If a and b are two rational numbers, then between these two numbers, one

rational number will be $\left(\frac{a+b}{2}\right)$

Required rational number between $\frac{3}{5}$ and $\frac{7}{9}$

$= \frac{1}{2}\left(\frac{3}{5} + \frac{7}{9}\right) = \frac{1}{2}\left(\frac{27+35}{45}\right) = \frac{1}{2} \times \frac{62}{45} = \frac{31}{45} \therefore \frac{3}{5} < \frac{31}{45} < \frac{7}{9}$

(ii) Required rational number between 8 and 8.04

$= \frac{1}{2}(8 + 8.04) = \frac{1}{2}(16.04) = 8.02 \therefore 8 < 8.02 < 8.04$

4. Insert three rational numbers between

(i) 4 and 5

(ii) $\frac{1}{2}$ and $\frac{3}{5}$

(iii) 4 and 4.5

(iv) $2\frac{1}{3}$ and $3\frac{2}{3}$

(v) $-\frac{1}{2}$ and $\frac{1}{3}$

Solution:

(i) The given numbers are 4 and 5.

As, $4 < 5$

$\Rightarrow 4 < \frac{1}{2}\left(\frac{4+5}{1}\right) < 5 \Rightarrow 4 < \frac{9}{2} < 5$

$\Rightarrow 4 < 4.5 < 5$ ... (1)

Again, $4 < \frac{1}{2}\left(4 \div \frac{9}{2}\right) < \frac{9}{2}$

$\Rightarrow 4 < 4.25 < 4.5$ ... (2)

Again, $4.5 < 5 \Rightarrow 4.5 < \frac{1}{2}(4.5+5) < 5 \Rightarrow 4.5 < 4.75 < 5$ ... (3)

$\therefore$ From equation (1), (2) and (3), we get $4 < 4.25 < 4.5 < 4.75 < 5.$

Thus, required rational numbers between 4 and 5 are 4.25, 4.75 and 4.5

(ii) The given numbers are $\frac{1}{2}$ and $\frac{3}{5}$

As, $\frac{1}{2} < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{1}{2}+\frac{3}{5}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{5+6}{10}\right) < \frac{3}{5}$

$\Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{11}{10}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{11}{20} < \frac{3}{5}$

Again, $\frac{1}{2} < \frac{1}{2}\left(\frac{1}{2} \div \frac{11}{20}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{21}{20}\right) < \frac{3}{5}$

$\frac{1}{2} < \frac{21}{40} < \frac{3}{5}$ ... (2)

Again, $\frac{11}{20} < \frac{3}{5} \Rightarrow \frac{11}{20} < \frac{1}{2}\left(\frac{11}{20} \div \frac{3}{5}\right) < \frac{3}{5} \Rightarrow \frac{1}{2} < \frac{1}{2}\left(\frac{23}{20}\right) < \frac{3}{5}$

$\Rightarrow \frac{1}{2} < \frac{23}{40} < \frac{3}{5}$ ... (3)

From equation (1), (2) and (3), we get

$\frac{1}{2} < \frac{21}{40} < \frac{11}{20} < \frac{23}{40} < \frac{3}{15}$

Thus, required rational numbers between $\frac{1}{2}$ and $\frac{3}{5}$ are $\frac{21}{40}, \frac{11}{20}$ and $\frac{23}{40}$

(iii) The given numbers are 4 and 5

As $4 < 4.5 \Rightarrow 4 < \frac{1}{2}(4 \div 4.5) < 4.5$

$\Rightarrow 4 < 4.25 < 4.5$ ... (1)

$\Rightarrow 4 < \frac{1}{2}(4 + 4.25) < 4.25 \Rightarrow 4 < 4.125 < 4.25$ ... (2)

Again, $4.25 < 4.5$

$\Rightarrow 4.25 < \frac{1}{2}(4.25 \div 4.5) \div 4.5 \Rightarrow 4.25 < 4.375 < 4.5$

From equations (1), (2) and (3), we have $4 < 4.125 < 4.25 < 4.375 < 4.5$

Thus, required rational numbers between 4 and 4.5 are 4.125, 4.25 and 4.375

(iv) The given numbers are $2\frac{1}{3}$ and $3\frac{2}{3}$ i.e., $\frac{7}{3}$ and $\frac{11}{3}$

As $\frac{7}{3} < \frac{11}{3} \Rightarrow \frac{7}{3} < \frac{1}{2}\left(\frac{7}{3} \div \frac{11}{3}\right) < \frac{11}{3}$

$\Rightarrow \frac{7}{3} < \frac{1}{2}\left(\frac{18}{3}\right) < \frac{11}{3} \Rightarrow \frac{7}{3} < \frac{18}{6} < \frac{11}{3}$

$\Rightarrow \frac{7}{3} < 3 < \frac{11}{3}$ ... (1)

Again, $\frac{7}{3} < \frac{1}{2}\left(\frac{7}{3} \div \frac{3}{1}\right) < 3$

$\frac{7}{3} < \frac{8}{3} < 3$ ... (2)

Again, $3 < \frac{11}{3}$

$3 < \frac{1}{2}\left(3 + \frac{11}{3}\right) < \frac{11}{3} \Rightarrow 3 < \frac{1}{2}\left(\frac{20}{3}\right) < \frac{11}{3}$

$3 < \frac{10}{3} < \frac{11}{3}$ ... (3)

From equation (1), (2) and (3), we get

$\frac{7}{3} < \frac{8}{3} < 3 < \frac{10}{3} < \frac{11}{3}$

Thus, the required numbers between $2\frac{1}{3}$ and $3\frac{2}{3}$ i.e., $\frac{7}{3}$ and $\frac{11}{3}$ are $\frac{8}{3}$, 3 and $\frac{10}{3}$

5. State, whether the following numbers are rational or irrational:

(i) $\left(2 + \sqrt{2}\right)^2$

(ii) $\left(5 + \sqrt{5}\right)\left(5 - \sqrt{5}\right)$

Solution:

(i) $\left(2+\sqrt{2}\right)^2 = 4 + 2 + 2 \times 2 \times \sqrt{2} = 6 + 4\sqrt{2}$

Hence, it is an irrational number.

(ii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right) = (5)^2 = \left(\sqrt{5}\right)^2$

$= 25 - 5 = 20$ Hence, it is a rational number.

6. Use division method to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

Solution:

$\sqrt{3} = 1.73205...$

	1.73205
1	3.00 00 00 00 00
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
346405	1760000
	1732025
	28975

It is non-terminating and non-recurring decimals.

$\therefore$ $\sqrt{3}$ is an irrational number.

$\sqrt{5} = 2.2360679...$

	2.2360679
2	5.00 00 00 00 00 00
	1
42	100
	84
443	1600
	1329
4466	27100
	26796
447206	3040000
	2683236
4472127	35676400
	31304889
44721349	437151100
	402492141
	34658959

It is non terminating and non recurring decimals.

$\therefore$ $\sqrt{5}$ is an irrational number.

7. Show that:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^{2}$ **is rational**

(ii) $\left(\sqrt{3}+\sqrt{5}\right)$ **is an irrational number**

Solution:

(i) Let $\left(\sqrt{3}+\sqrt{7}\right)$ is a rational number.

Then square of given number i.e., $\left(\sqrt{3}+\sqrt{7}\right)^{2}$ is rational.

$\Rightarrow \left(\sqrt{3}+\sqrt{7}\right)^{2}$ is rational

$\Rightarrow \left(\sqrt{3}+\sqrt{7}\right)^{2} \div 2\sqrt{3}\times\sqrt{7} = 3 \div 7 + 2\sqrt{21} = \left(10+2\sqrt{21}\right)$ is rational.

But, $\left(10+2\sqrt{21}\right)$ being the sum of a rational and irrational is irrational. This contradiction arises by assuming that $\left(\sqrt{3}+\sqrt{7}\right)$ is rational number.

Hence, $\left(\sqrt{3}+\sqrt{7}\right)$ is an irrational number.

(ii) Let $\left(\sqrt{3}+\sqrt{5}\right)$ is a rational number.

Then square of given number i.e., $\left(\sqrt{3}+\sqrt{5}\right)^{2}$ is rational.

$\Rightarrow \left(\sqrt{3}+\sqrt{5}\right)^{2}$ is rational

$\Rightarrow \left(\sqrt{3}\right)^{2}+\left(\sqrt{5}\right)^{2}+2\sqrt{3}\times\sqrt{5} = 3+5+2\sqrt{15} = \left(8+2\sqrt{15}\right)$ rational.

But, $\left(8+2\sqrt{15}\right)$ being the sum of a rational and irrational it is irrational.

This contradiction arises by assuming that $\left(\sqrt{3}+\sqrt{5}\right)$ is rational.

Hence, $\left(\sqrt{3}+\sqrt{5}\right)$ is irrational number.

8. Insert three irrational numbers between 0 and 1.

Solution:

Three irrational numbers between 0 and 1 can be

$0 < 0.1011001110001111\ldots < 0.1010011000111\ldots < 0.202002000200002\ldots < 1$

9. If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, find the value of a and b.

Solution:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$$

Multiplying both sides numerator and denominator of L.H.S by $(\sqrt{3}-1)$, we get

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} \Rightarrow \frac{3+1-2\sqrt{3}\times 1}{3-1} \Rightarrow \frac{4-2\sqrt{3}}{2} \Rightarrow 2-\sqrt{3}$$

But $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, so $2-\sqrt{3} = a + b\sqrt{3}$

Comparing both sides

$\Rightarrow a = 2$ and $b = -1$

10. If $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, find the value of a and b.

Solution:

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

Multiplying numerator and denominator of L.H.S. by $3 \div \sqrt{2}$, we get

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{(3+\sqrt{2})(3\div\sqrt{2})}{(3-\sqrt{2})(3\div\sqrt{2})}$$

$$\Rightarrow \frac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2} \Rightarrow \frac{9+2+2\times 3\sqrt{2}}{9-2} \Rightarrow \frac{11\div 6\sqrt{2}}{7} \Rightarrow \frac{11}{7}+\frac{6\sqrt{2}}{7}$$

But $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, so $\frac{11}{7}+\frac{6\sqrt{2}}{7} = a \div b\sqrt{2}$

Comparing both sides, $a = \frac{11}{7}, b = \frac{6}{7}$

9. Simplify:

(i) $\dfrac{\mathbf{22}}{\mathbf{2\sqrt{3}+1}}+\dfrac{\mathbf{17}}{\mathbf{2\sqrt{3}-1}}$

(ii) $\dfrac{\sqrt{\mathbf{2}}}{\sqrt{\mathbf{6}}-\sqrt{\mathbf{2}}}-\dfrac{\sqrt{\mathbf{3}}}{\sqrt{\mathbf{6}}+\sqrt{\mathbf{2}}}$

(iii) $\dfrac{\mathbf{18}}{\mathbf{3\sqrt{2}-2\sqrt{3}}}+\dfrac{\mathbf{1}}{\mathbf{3\sqrt{2}+2\sqrt{3}}}$

Solution:

(i) $\dfrac{22}{2\sqrt{3}+1}+\dfrac{17}{2\sqrt{3}-1}$

By rationalising the denominator of each term, we get

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}=\frac{22}{2\sqrt{3}+1}\times\frac{2\sqrt{3}-1}{2\sqrt{3}-1}+\frac{17}{2\sqrt{3}-1}\times\frac{2\sqrt{3}+1}{2\sqrt{3}+1}$$

$$=\frac{44\sqrt{3}-22}{\left(2\sqrt{3}\right)^2-(1)^2}+\frac{34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{44\sqrt{3}-22}{12-1}+\frac{34\sqrt{3}\div 17}{12-1}=\frac{44\sqrt{3}-22}{11}\div\frac{34\sqrt{3}\div 17}{11}$$

$$=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{11}=\frac{78\sqrt{3}-5}{11}$$

(ii) $\dfrac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\dfrac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

By rationalising the denominator of each term, we get

$$\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$=\frac{\sqrt{2\times 6}+2}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}-\frac{\sqrt{3\times 6}-\sqrt{6}}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{2\times 2\times 3}+2}{6-2}-\frac{\sqrt{3\times 3\times 2}-\sqrt{6}}{6-2}$$

$$=\frac{2\sqrt{3}+2-\left(3\sqrt{2}-\sqrt{6}\right)}{4}=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

(iii) $\dfrac{18}{3\sqrt{2}-2\sqrt{3}}+\dfrac{1}{5\sqrt{2}+2\sqrt{3}}$

By rationalising the denominator of each term, we get

$$\frac{18}{3\sqrt{2}-2\sqrt{3}}+\frac{1}{5\sqrt{2}+2\sqrt{3}}=\frac{18}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{5\sqrt{2}+2\sqrt{3}}\times\frac{5\sqrt{2}-2\sqrt{3}}{5\sqrt{2}-2\sqrt{3}}$$

$$=\frac{54\sqrt{2}+36\sqrt{3}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{5\sqrt{2}-2\sqrt{3}}{\left(5\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$=\frac{54\sqrt{2}+36\sqrt{3}}{18-12}+\frac{5\sqrt{2}-2\sqrt{3}}{38}$$

$$=\frac{19\left(54\sqrt{2}+36\sqrt{3}\right)+3\left(5\sqrt{2}-2\sqrt{3}\right)}{114}$$

$$=\frac{1026\sqrt{2}+684\sqrt{3}+15\sqrt{2}-6\sqrt{3}}{114}$$

$$=\frac{1041\sqrt{2}+678\sqrt{3}}{114}=\frac{1041\sqrt{2}}{114}+\frac{678\sqrt{3}}{114}$$

$$=\frac{347\sqrt{2}}{38}+\frac{113\sqrt{3}}{19}$$

10. If $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y=\frac{\sqrt{5}+2}{\sqrt{5}-2}$; find: x^2+y^2+xy

(i) x^2

(ii) y^2

(iii) xy

(iv) x^2+y^2+xy

Solution:

(i) $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}=\frac{\sqrt{5}-2}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$

$x=\frac{\left(\sqrt{5}-2\right)^2}{5-4}=\frac{5+4-4\sqrt{5}}{1}=9-4\sqrt{5}$

$\therefore x=9-4\sqrt{5}$

Squaring both sides, we get

$\Rightarrow x^2=\left(9-4\sqrt{5}\right)^2=81+16(5)-72\sqrt{5}=81+80-72\sqrt{5}=161-72\sqrt{5}$

(ii) $y=\frac{\sqrt{5}+2}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{5+4+4\sqrt{5}}{5-4}=9+4\sqrt{5}$

$y = 9 + 4\sqrt{5}$

Squaring both sides, we get

$\therefore y^2 = \left(9 + 4\sqrt{5}\right)^2 = 81 + 80 + 2 \times 9 \times 4\sqrt{5} = 161 + 72\sqrt{5}$

(iii) $xy = \left(9 + 4\sqrt{5}\right)\left(9 + 4\sqrt{5}\right) = 81 - 80 = 1$

(iv) $x^2 + y^2 + xy = 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1 = 323$

11. Write down the values of:

(i) $\left(\frac{3}{2}\sqrt{2}\right)^2$

(ii) $\left(5 + \sqrt{3}\right)^2$

(iii) $\left(\sqrt{6} - 3\right)^2$

(iv) $\left(\sqrt{5} + \sqrt{6}\right)^2$

Solution:

(i) $\left(\frac{3}{2}\sqrt{2}\right)^2 = \frac{3}{2}\sqrt{2} \times \frac{3}{2}\sqrt{2} = \frac{9}{4}\left(\sqrt{2}\right)^2 = \frac{9}{4} \times 2 = \frac{9}{2}$

(ii) $\left(5 + \sqrt{3}\right)^2 = (5)^2 + \left(\sqrt{3}\right)^2 + 2(5)\left(\sqrt{3}\right)$ [using $(a + b)^2 = a^2 + b^2 + 2ab$]

$= 25 + 3 + 10\sqrt{3}$

$= 28 + 10\sqrt{3}$

(iii) $\left(\sqrt{6} - 3\right)^2 = \left(\sqrt{6}\right)^2 + (3)^2 - 2 \times \sqrt{6} \times 3$ [using $(a - b)^2 = a^2 + b^2 - 2ab$]

$= 6 + 9 - 6\sqrt{6} = 15 - 6\sqrt{6}$

(iv) $\left(\sqrt{5} + \sqrt{6}\right) = \left(\sqrt{5}\right)^2 + \left(\sqrt{6}\right)^2 + 2 \times \sqrt{5} \times \sqrt{6}$ [using $(a + b)^2 = a^2 + b^2 + 2ab$]

$= 5 + 6 + 2\sqrt{30} = 11 + 2\sqrt{30}$

12. Rationalize the denominator of:

(i) $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

(ii) $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}}$

(iii) $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

Solution:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Multiplying numerator and denominator by $\sqrt{3}-\sqrt{2}$, we get

$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{(\sqrt{3})^2+(\sqrt{2})^2-2\times\sqrt{3}\times\sqrt{2}}{3-2}$$

$$=\frac{3+2-2\sqrt{6}}{1}=5-2\sqrt{6}$$

(ii) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

Multiplying numerator and denominator by $\sqrt{7}+\sqrt{5}$, we get

$$=\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{(\sqrt{7}+\sqrt{5})^2}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$=\frac{(\sqrt{7})^2+(\sqrt{5})^2+2\times\sqrt{7}\times\sqrt{5}}{7-5}$$

$$=\frac{7+5+2\sqrt{35}}{2}=\frac{12+2\sqrt{35}}{2}$$

$$=\frac{2(6+\sqrt{35})}{2}=6+\sqrt{35}$$

(iii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Multiplying numerator and denominator by $\sqrt{5}+\sqrt{3}$, we get

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{(\sqrt{5})^2+(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}}{5-3}$$

$$=\frac{5+3+2\sqrt{15}}{2}=\frac{8+2\sqrt{15}}{2}=\frac{2(4+\sqrt{15})}{2}=4+\sqrt{15}$$

13. Find the values of 'a' and 'b' in each of the following:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Solution:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

Multiplying numerator and denominator of L. H. S. by $(2+\sqrt{3})$, we get

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{4-3}$$

$$= \frac{(2)^2 + (\sqrt{3})^2 + 2\times 2\times\sqrt{3}}{1} \qquad \{\text{using } (a+b)^2 = a^2 + 2a + b^2\}$$

$$= 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$$

But $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$. So, $7 + 4\sqrt{3} = a + b\sqrt{3}$

Comparing both sides we get :

$a = 7$ and $b = 4$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

Multiplying numerator and denominator of L. H. S. by $\sqrt{7}-2$, we get

$$\frac{\sqrt{7}-2}{\sqrt{7}+2} = \frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2} = \frac{(\sqrt{7}-2)^2}{7-4}$$

$$= \frac{(\sqrt{7})^2 + (2)^2 - 2\times 2\times\sqrt{7}}{3} \qquad \{\text{using } (a-b)^2 = a^2 - 2ab + b^2\}$$

$$= \frac{7+4-4\sqrt{7}}{3} = \frac{11-4\sqrt{7}}{3}$$

But $\frac{\sqrt{7-2}}{\sqrt{7+2}} = a\sqrt{7} + b$. So, $\frac{11}{3} - \frac{4\sqrt{7}}{3} = a\sqrt{7} + b$

Comparing both sides, we get

$a\sqrt{7} = \frac{-4\sqrt{7}}{3}$ and $b = \frac{11}{3}$

$\Rightarrow a = \frac{-4}{3}$ and $b = \frac{11}{3}$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

Multiplying numerator and denominator of L. H. S. by $\sqrt{3}+\sqrt{2}$, we get

$$\frac{3}{\sqrt{3}-\sqrt{2}} = \frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{3\sqrt{3}+3\sqrt{2}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{3\sqrt{3}+3\sqrt{2}}{3-2} = \frac{3\sqrt{3}+3\sqrt{2}}{1}$$

Also, $\frac{3}{\sqrt{3}-\sqrt{2}} - a\sqrt{3} - b\sqrt{2}$. So, $3\sqrt{3}+3\sqrt{2} = a\sqrt{3} - b\sqrt{2}$

Comparing both sides, we get $a = 3$ and $b = -3$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Multiplying numerator and denominator of L. H. S. by $5+3\sqrt{2}$, we get

$$\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = \frac{5+3\sqrt{2}}{5-3\sqrt{2}} \times \frac{5+3\sqrt{2}}{5+3\sqrt{2}} = \frac{\left(5+3\sqrt{2}\right)^2}{(5)^2 - \left(3\sqrt{2}\right)^2}$$

$$= \frac{(5)^2 + \left(3\sqrt{2}\right)^2 + 2 \times 5 \times 3\sqrt{2}}{25-18} \qquad \{\text{using } (a+b)^2 = a^2 + 2ab + b^2\}$$

$$= \frac{25+18+30\sqrt{2}}{7} = \frac{43+30\sqrt{2}}{7}$$

Also, $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$. So, $\frac{43}{7} + \frac{30\sqrt{2}}{7} = a + b\sqrt{2}$

|Comparing both sides, we get: $a = \frac{43}{7}$ and $b = \frac{30}{7}$