Class – IX	Topic – Rectilinear figure

1. The angles of a quadrilateral are in the ratio 3:2:4:1. Find the angles. Assign a special name to the quadrilateral.

Solution:

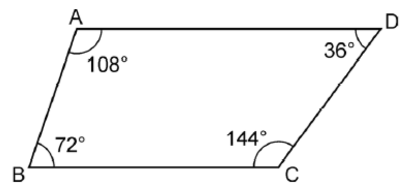
The ratio of angles of quadrilateral $= 3 : 2 : 4 : 1$

Let, the angle of quadrilateral $= 3x°, 2x°, 4x°, 1x°$

Sum of angles of a quadrilateral $= 360°$

$\Rightarrow 3x° + 2x° + 4x° + x° = 360°$

$\Rightarrow 10x^2 = 360° \Rightarrow 10x = 360 \Rightarrow x = \frac{360°}{10} \Rightarrow x = 36°$



$\therefore$ Angles of quadrilateral are $3x°, 2x°, 4x°, 1x°$

$\Rightarrow$ Angles of quadrilateral are $3 \times 36°, 2 \times 36°, 4 \times 36°, 1 \times 36°$

$\Rightarrow$ Angles of quadrilateral are $108°, 72°, 144°, 36°$

In the adjoining figure

$\angle A + \angle B = 108° + 72° \Rightarrow \angle A + \angle B = 180°$

i. e., Sum of interior angles on the same side of transversal AB $= 180°$

$\therefore AD || BC$.

Hence, quadrilateral ABCD is a trapezium.

2. In the adjoining figure, equilateral ΔEDC surmounts square ABCD. If $\angle$DEB $= x°$, find he value of x.

Solution:

From figure, ABCD is a square and ΔCDE is an equilateral triangle. BE is joined, $\angle DEB = x°$

In ΔBCE, BC = CE = CD

$\therefore \angle CBE = \angle CEB$

and $\angle BCE = \angle BCD + \angle DCE = 90° + 60° = 150°$

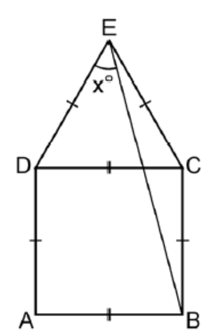
But $\angle BCE + \angle CBE + \angle CEB = 180°$ (Sum of a triangle is 180°)

$\Rightarrow 150° + \angle CEB + \angle CEB = 180°$

$\Rightarrow 150° + 2\angle CEB = 180°$

$\Rightarrow 2\angle CEB = 180° - 150° = 30°$

$\therefore \angle CEB = \frac{30°}{2} = 15°$



But $\angle CED = 60°$ (Angle of an equilateral triangle is 60°)

$\Rightarrow x^\circ + \angle CEB = 60^\circ$

$\Rightarrow x^\circ + 15^\circ = 60^\circ$

$\Rightarrow x^\circ = 60^\circ - 15^\circ = 45^\circ$

$\therefore x = 45^\circ$

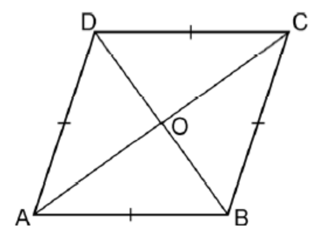
3. In the adjoining figure, ABCD is a rhombus whose diagonals intersect at O. If $\angle OAB : \angle OBC = 2 : 3$, find the angles of ΔOAB.

Solution:

ABCD is a rhombus and its diagonal bisect each other at right angles at O.

$\angle OAB : \angle OBA = 2 : 3$

Let $\angle OAB = 2x$ and $\angle OBA = 3x$



But $\angle AOB = 90^\circ$

$\therefore \angle OAB + \angle OBA = 90^\circ$

$\Rightarrow 2x + 3x = 90^\circ$

$\Rightarrow 5x = 90^\circ$

$\therefore x = \dfrac{90^\circ}{5} = 18^\circ$

$\therefore \angle OAB = 2x = 2 \times 18^\circ = 36^\circ$

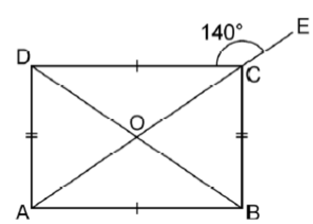
$\angle OBA = 3x = 3 \times 18^\circ = 54^\circ$ and $\angle AOB = 90^\circ$

4. In the adjoining figure, ABCD is a rectangle whose diagonals intersect at O. diagonal AC is produced to E and $\angle ECD = 140^\circ$. Find the angles of ΔOAB.

Solution:

ABCD is a rectangle and its diagonals AC and BD bisect each other at O.

Diagonal AC is produced to E such that $\angle ECD = 140^\circ$



$\angle ECD + \angle DCO = 180^\circ$ (Linear pair)

$\Rightarrow 140^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 140^\circ = 40^\circ$

But $OC = OD$ (Half of equal diagonals)

$\therefore CDO = \angle DCO = 40^\circ$

Now $\because AB || CD$ (Opposite sides of a rectangle)

$\therefore \angle OAB = \angle DCO = 40^\circ$ (Alternate angles)

Similarly, $\angle OBA = 40^\circ$

In $\Delta AOB, \angle OBA + \angle OAB + \angle AOB = 180^\circ$ (Sum of angles of atriangle is 180°)

$\Rightarrow 40° + 40° + \angle AOB = 180°$

$\Rightarrow 80° + \angle AOB = 180°$

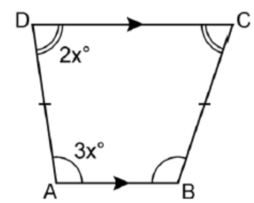
$\Rightarrow \angle AOB = 180° - 80° = 100°$

5. **In the given adjoining figure, ABCD is an isosceles trapezium in which $\angle CDA = 2x°$ and $\angle BAD = 3x°$. Find all the angles of the trapezium.**

Solution:

ABCD is an isosceles trapezium in which $AD = BC$ and $AB || CD$.

$\angle BAD + \angle CDA = 180°$ (Co – interior angles)



$\Rightarrow 3x + 2x = 180°$

$\Rightarrow 5x = 180°$

$\therefore x = \frac{180°}{5} = 36°$

$\therefore \angle A = 3x = 3 \times 36° = 108°, \angle D = 2x = 2 \times 36° = 72°$

$\because$ ABCD is an isosceles trapezium.

$\therefore \angle A = \angle B$ and $\angle C = \angle D$

$\therefore \angle B = 108°$ and $\angle C = 72°$

Hence, $\angle A = 108°, \angle B = 108°, \angle C = 72°, \angle D = 72°$

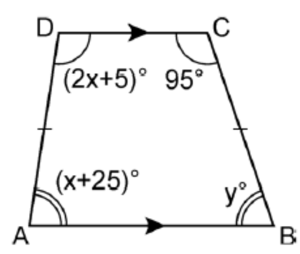
6. **In the given figure, ABCD is trapezium in which $\angle A = (x + 25)°, \angle B = y°, \angle C = 95°$ and $\angle D = (2x + 5)°$. Find the values of x and y.**

Solution:

In a trapezium ABCD

$\angle A = (x + 25)°, \angle B = y°, \angle C = 95°$ and $\angle D = (2x + 5)°$

$\angle A + \angle D = 180°$ (Co – interior angles)



$\Rightarrow (x + 25)° + (2x + 5)° = 180°$

$\Rightarrow x + 25° + 2x + 5° = 180°$

$\Rightarrow 3x + 30 = 180°$

$\Rightarrow 3x = 180° - 30° = 150°$

$\therefore x = \frac{150°}{3} = 50°$

Similarly, $\angle B + \angle C = 180°$

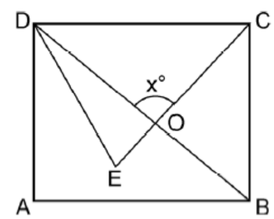
$\Rightarrow y + 95° = 180° \Rightarrow y = 180° - 95° = 85°$.

Hence, $x = 50°, y = 85°$.

7. DEC is an equilateral triangle in a square ABCD. If BD and CE intersect at O and $\angle COD = x°$, find the value of x.

Solution:

ABCD is a square and ΔECD is an equilateral triangle. Diagonal BD and CE intersect each other at O, $\angle COD = x°$.



$\because$ BD is the diagonal of square ABCD

$\therefore \angle BDC = \frac{90°}{2} = 45° \Rightarrow \angle ODC = 45°$

$\angle ECD = 60°$ (Angle of equilateral triangle) or $\angle OCD = 60°$

Now in ΔOCD,

$\angle OCD + \angle ODC + \angle COD = 180°$ (Sum of angles of a triangle is 180°)

$\Rightarrow 45° + 60° + x° = 180°$

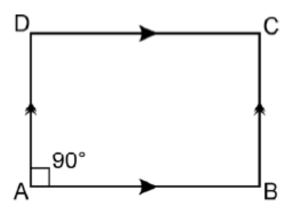
$\Rightarrow 105° + x° = 180°$

$\therefore x° = 180° - 105° = 75°$

Hence, $x = 75°$

8. If one angle of a parallelogram is $90°$, show that each of its angles measure $90°$.

Solution:



Given : ABCD is a parallelogram and $\angle A = 90°$

To Prove : Each angle of the parallelogram ABCD is 90°.

Proof : In parallelogram ABCD,

$\because \angle A = \angle C$

$\therefore \angle C = 90°$ ($\because \angle A = 90°$)

But $\angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - 90° = 90°$ and $\angle B = \angle D$ (Opposite angles of a parallelogram)

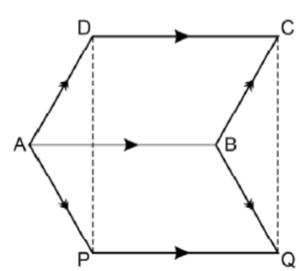
$\therefore \angle B = 90°$. Hence, $\angle B = \angle C = \angle D = 90°$

9. In the adjoining figure, ABCD and PQBA are two parallelograms. Prove that :

(i) DPQC is a parallelogram

(ii) DP = CQ

(iii) $\Delta DAP \cong \Delta CBQ$



Solution:

Given: ABCD and PQBA are two parallelogram PD and QC are joined.

(i) ABCD and PQBA are parallelogram

$DC||AB$ and $AB||PQ$ (Given)

$\therefore DC||PQ$

Again $DC = AB$ and $AB = PQ$ (Opposite sides of parallelograms)

$\therefore DC = PQ$

$\because DC = PQ$ and $DC||PQ$

$\therefore$ DPQC is a parallelogram.

(ii) $\therefore DP = CQ$ (Opposite sides of a parallelogram)

(iii) In ΔDAP and ΔCBQ

$DA = CB$ (Opposite sides of a parallelogram)

$AP = BQ$ (Opposite sides of a parallelogram)

$PD = CQ$

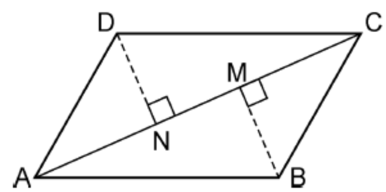
$\therefore \Delta DAP \cong \Delta CBQ$ (SSS axiom of congruency)

Hence Proved.

10. In the adjoining figure, ABCD is a parallelogram. $BM \perp AC$ and $DN \perp AC$. Prove that :

(i) $\Delta BMC \cong \Delta DNA$

(ii) $BM = DN$



Solution:

Given : ABCD is a parallelogram.

$BM \perp AC$ and $DN \perp AC$.

Proof : In ΔBMC and ΔDNA

$BC = AD$ (Opposite sides of a parallelogram)

$\angle M = \angle N = 90°$

$\angle BCM = \angle DAN$ (Alternate angles)

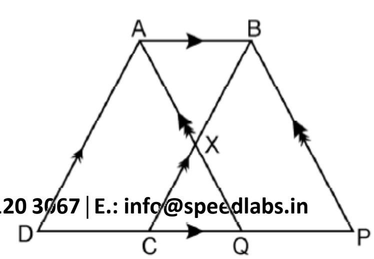
(i) $\therefore \Delta BMC \cong \Delta DNA$ (AAS axiom of congruency)

(ii) $\therefore BM = DN$ (C. P. C. T)

11. In the adjoining figure, ABCD is a parallelogram and X is the mid-point of BC. The line AX produced meets DC produced at Q. the parallelogram AQPB is completed. Prove that :

(i) $\Delta ABX \cong \Delta QCX$

(ii) $DC = CQ = QP$



Solution:

Given : ABCD is a parallelogram. X is the mid-point of BC.

AX is joined and produced to meet DC at Q. From B, BP is drawn parallel to AQ so that AQPB is a parallelogram.

(i) In ΔABX and ΔQCX

$XB = XC$ ($\because$ X is the mid – point of BC)

$\angle AXB = \angle CXQ$ (Vertically opposite angles)

$\angle BAX = \angle XQC$ (Alternate angles)

$\therefore \Delta ABX \cong \Delta QCX$ (ASA axiom of congruency)

(ii) In parallelogram ABCD,

$AB = DC$ ... (1) (Opposite ides of a parallelogram)

Similarly, in parallelogram AQPB

$AB = QP$ ... (2)

$\therefore$ From equation (1) and (2), we get

$DC = QP$ ... (3)

In ΔBCP,

X is the mid – point of BC and $AQ || BP$

$\therefore$ Q is mid – point of CP.

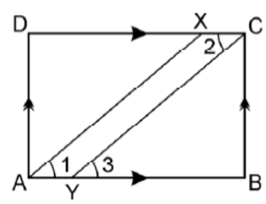
$\Rightarrow CQ = QP$ ... (4)

From equation (3) and (4), we get

$DC = QP = CQ$ or $DC = CQ = QP$.

12. In the adjoining figure, ABCD is a parallelogram. Line segment AX and CY bisect $\angle A$ and $\angle C$ respectively.

Prove that :



(i) $\Delta ADX \cong \Delta CBY$

(ii) $AX = CY$

(iii) $AX || CY$

(iv) AYCX is a parallelogram

Solution:

Given : ABCD is a parallelogram. Line segments AX and CY bisect $\angle A$ and $\angle C$ respectively.

(i) In ΔADX and ΔCBY

$AD = BC$ (Opposite sides of a parallelogram)

$\angle D = \angle B$ (Opposite angles of the parallelogram)

$\angle DAX = \angle BCY$ (Half of equal angles A and C)

$\therefore \Delta ADX \cong \Delta CBY$ (ASA axiom of congruency)

(ii) $\therefore AX = CY$ (C. P. C. T)

(iii) $\angle 1 = \angle 2$ (Half of equal angles)

But $\angle 2 = \angle 3$ (alternate angles)

$\therefore \angle 1 = \angle 3$

But these are corresponding angles.

$\therefore AX || CY$

(iv) $\because AX = CY$ and $AX || CY$

$\therefore$ AYCX is a parallelogram

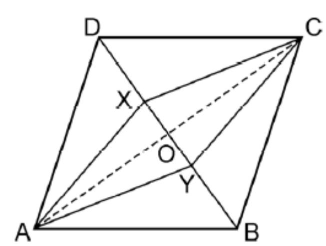
13. In the adjoining figure, ABCD is a parallelogram and X, Y are points on diagonal BD such that DX=BY. Prove that CXAY is a parallelogram.

Solution:

Given : ABCD is a parallelogram, X and Y are points on diagonal BD such that $DX = BY$.

Construction : Join AC meeting BD at O.

Proof : $\therefore$ AC and BD are the diagonals of the parallelogram ABCD.



$\therefore$ AC and BD bisect each other at O.

$\therefore AO = OC$ and $BO = OD$

But $DX = BY$ (Given)

$\therefore DO - DX = OB - BY$

$\Rightarrow OX = OY$

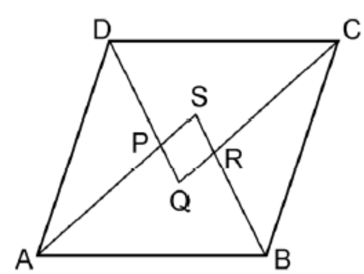
Now in quadrilateral CXAY, diagonals AC and XY bisect each other at O.

$\therefore$ CSAY is a parallelogram.

14. Show that the bisectors of the angles of a parallelogram enclose a rectangle.

Solution:

Given : ABCD is a parallelogram.



Bisectors of $\angle A$ and $\angle B$ meet at S and bisectors of $\angle C$ and $\angle D$ meet at Q.

$\because \angle A + \angle B = 180°$

$\therefore \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

$\Rightarrow \angle SAB = \angle SBA = 90°$

Similarly we can prove that $\angle CQD = 90°$

SpeedLabs
Assisted Practice | Expert Guidance | Personalized Insights
An Innovative Learning Methodology by IITians.

Again, $\angle A + \angle D = 180°$

$\therefore \frac{1}{2}\angle A + \frac{1}{2}\angle D = 90°$

$\Rightarrow \angle PAD = \angle PDA = 90°$

$\therefore \angle APD = 90°$

But $\angle SPQ = \angle APD$ (Vertically opposite angles)

$\therefore \angle SPQ = 90°$

$\because$ Similarly, we can prove that $\angle SRQ = 90°$

$\because$ In quadrilateral PQRS, its each angle is of 90°.

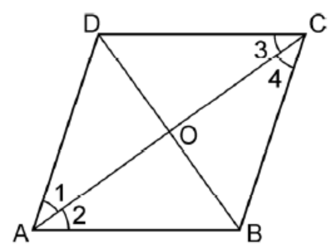
Hence, PQRS is a rectangle.

15. If a diagonal of a parallelogram bisects one of the angles of the parallelogram, prove that it also bisects the second angle and then the two diagonals are perpendicular to each other.

Solution:

Given : In parallelogram ABCD, diagonal AC bisects $\angle A$, BD is joined meeting AC at O.

Proof : In parallelogram ABCD



$\because AB || DC$

$\therefore \angle 1 = \angle 4$

and $\angle 2 = \angle 3$ (Alternate angles)

But $\angle 1 = \angle 2$ (Given)

$\therefore \angle 3 = \angle 4$

Hence, AC bisects $\angle C$ also. Similarly we can prove that diagonal BD will also bisect the $\angle B$ and $\angle D$.

$\therefore$ ABCD is a rhombus.

But diagonals of a rhombus bisect each other at right angles.

$\therefore$ AC and BD are perpendicular to each other.

1.