SpeedLabs

MATHS

ICSE 8th

TEEVRA EDUTECH PVT. LTD.

1. Which of the following collections of objects are sets?

(i) All the months in a year

Ans. YES. It is a well-defined collection of distinct objects. There are 12 months in a year. So, it is a definite set of elements.

A = {January, February, March, April, May, June, July, August, September, October, November, December}

(ii) All the planets in our solar system

Ans. YES. It is a well-defined collection of distinct objects.

A= {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}

2. Describe the following sets in roster form

(i) $B = \{x \mid x \in W, x <= 6\}$

Ans. $B = \{0, 1, 2, 3, 4, 5, 6\}$

(ii) $C = \{x \mid x$ is a factor of $32\}$

Ans. $C = \{1, 2, 4, 8, 16, 32\}$

3. Describe the following sets in set builder form

(i) $A = \{5, 6, 7, 8, 9, 10, 11, 12\}$

Ans. $A = \{x \mid x \in N, 4 < x < 13\}$ or $A = \{x \mid x \in N, 5 <= x <=12\}$

(ii) $B = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

Ans. $B = \{x \mid x$ is a factor of $48\}$

4. Which of the following are empty sets?

(i) $A = \{x \mid x \in N, x + 5 = 5\}$

Ans. Empty Set

Note: Natural Number N = {1, 2, 3, 4, 5...}. Since x+5 = 5 is given, it means, x = 0. But 0 does not belong to N and hence the set it empty or null.

(ii) $B = \{x \mid x \in N, 2x + 3 = 6\}$

Ans. Empty Set

Note: Natural Number N = {1, 2, 3, 4, 5...}. Since 2x+3 = 6 is given, it means, x = 1.5. But 1.5 does not belong to N and hence the set it empty or null.

Which of the following collections of objects are sets?

5. Rewrite the following statements using the set notations:

(i) p is an element of A

Ans. $p \in A$

(ii) q does not belong to set B

Ans. $q \notin B$

6. Separate finite and infinite sets from the following:

(i) Set of leaves on a tree

Ans. Finite. This is because in this case, the process of counting the leaves would surely come to an end.

(ii) $\{x \mid x \in N, x > 1000\}$

Ans. Infinite. This is because there is no end to the numbers since $x > 1000$.

7. Let A – {a, b, c, d}, B = {b, c, e} and C = {a, b, e}. Find:

Ans. 1. $A \cup B = \{a, b, c, d, e\}$

2. $B \cup C = \{a, b, c, e\}$

3. $A \cup C = \{a, b, c, d, e\}$

4. $A \cap B = \{b, c\}$

5. $B \cap C = \{b, e\}$

6. $A \cap C = \{a, b\}$

Note:

The union of sets A and B, denoted by $A \cup B$, is the set of all those elements, each one of which is either in A or in B or in both A and B

If there is a set $A = \{2, 3\}$ and $B = \{a, b\}$, then $A \cup B = \{2, 3, a, b\}$

So if $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ then

$x \in A \cup B$ which means $x \in A$ or $x \in B$

And if $x \notin A \cup B$ which means $x \notin A$ or $x \notin B$

The intersection of sets A and B is denoted by $A \cap B$, and is a set of all elements that are common in sets A and B.

If $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$, then $A \cap B = \{2\}$ as 2 is the only common element.

Thus $A \cap B = \{x: x \in A \text{ and } x \in B\}$

then $x \in A \cap B$ i.e. $x \in A$ and $x \in B$

And if $x \notin A \cap B$ i.e. $x \notin A$ and $x \notin B$

8. Let $A = \{1, 4, 7, 8\}$ and $B = \{4, 6, 8, 9\}$. Find i) $A - B$ and ii) $B - A$

Ans. $A - B = \{1, 4, 7, 8\} - \{4, 6, 8, 9\} = \{1, 7\}$

$B - A = \{4, 6, 8, 9\} - \{1, 4, 7, 8\} = \{6, 9\}$

Note:

For any two sets A and B, the difference $A - B$ is a set of all those elements of A which are not in B.

i.e. if A = {1, 2, 3, 4, 5} and B = {4, 5, 6}

Then A – B = {1, 2, 3} and B – A = {6}

Therefore, A – B = {x | x ∈ A and x ∉ B}, then

x ∈ A – B then x ∈ A but x ∉ B

9. Let ξ = {13, 14, 15, 16, 17, 18, 19, 20, 21}, A = {13, 17, 19} and B = {14, 16, 18, 20}. Find i) A′ and ii) B′

Ans. A′ = {14, 15, 16, 18, 20, 21}

B′ = {13, 15, 17, 19, 21}

Note:

Let x be the universal set and let A⊆ x. Then the complement of A, denoted by A′ is the set of all those elements of x which are not in A.

i.e. let ξ = {1, 2, 3, 4, 5,6 ,7 ,8} and A = {2, 3, 4}, then A′ = {1, 5, 6, 7, 8}

Thus A′ = {x | x ∈ ξ and x ∉ A} clearly x ∈ A′ and x ∉ A

Please note

ϕ′ = ξ and ϕ′= ξ

A ∪ A′ = ξ and A ∩ A′ = ϕ

10. Let ξ = {x | x ∈ Z, -4 ≤ x ≤ 4}, A = {x | x ∈ W, x <4} and B = {x | x ∈ N, 2 <x ≤ 4}. Find i) A′ and ii) B′

Ans. Note: First find out the elements of the three given sets.

ξ = {-4, -3, -2, -1, 0, 1, 2, 3, 4}, A = {0, 1, 2, 3} and B = {3, 4}

A′ = {-4, -3, -2, -1, 4}

B′ = {-4, -3, -2, -1, 0, 1, 2}

11. Let A = {b, c, d, e} and B = {d, e, f, g} be the two sub sets of the universal set ξ = {b, c, d, e, f, g}. Then verify the following:

(i) (A ∪ B)′ = (A′∩ B′)

Ans. LHS: A ∪ B = {b, c, d, e} ∪ {d, e, f, g} = {b, c, d, e, f, g}

Therefore, (A ∪ B)′ = ϕ

RHS: A′ = {f, g} and B′ = {b, c}

Therefore, A′∩ B′ = ϕ

Therefore, LHS = RHS. Hence proved.

(ii) (A ∩ B)′ = (A′∪ B′)

Ans. x = {b, c, d, e, f, g}

LHS: A ∩ B = {b, c, d, e} ∩ {d, e, f, g} = {d, e}

Therefore, (A ∩ B)′ = {b, c, f, g}

RHS: A′ = {f, g} and B′ = {b, c}

Therefore, A′∪ B′ = {b, c, f, g}

Therefore, LHS = RHS. Hence proved.

12. Let x = {x : x ∈ N, x < 50}, A = {x : x2 ∈ x}, B = {x : x = n2, n ∈ N} and C = {x : x is a factor of 36}. List all the elements of set A, B, and C. Also state whether each of the following statement is true or false.

Ans. ξ = {1, 2, 3, 4, ,5, …, 49}. Basically x is values from 1 to 49. Therefore:

A = {1, 2, 3, 4, 5, 6, 7}.

B = {1, 4, 9, 16, 25, 36, 49}

C = {1, 2, 3, 4, 6, 9, 12, 18, 36}. All factors of 36 but within 1 <= x <= 49

(i) $A \subseteq B$

Ans. False

(ii) A = B

Ans. False

(iii) $A \leftrightarrow B$

Ans. True since n(A) = 7 while n(B) = 7

(iv) $B \leftrightarrow C$

Ans. False since n(B) = 7 while n(C) = 9

(v) n(A) < n(C)

Ans. True since n(A) = 7 while n(C) = 9

13. Let A = {all triangles}, B = {all isosceles triangles} and C = {all equilateral triangles}. State, giving reasons, whether the following statements are true or false

(i) $B \subset C \subset A$

Ans. False

Note: All isosceles triangles are not equilateral triangles.

(ii) $C \subset B \subset A$

Ans. True

Note: All equilateral triangles are isosceles triangles and all isosceles triangles are triangles

14. Which of the following statements are correct?

(i) a ⊂ {a, b, c}

Ans. False

Note: a is an element of the set {a, b, c}. Hence it should be a ∈ {a, b, c}

(ii) {a} ⊂ {a, b, c}

Ans. Correct

Note: {a} is subset of the set {a, b, c}. Hence it should be a ⊂ {a, b, c}

(iii) φ ∈ {a, b, c}

Ans. False

Note: φ is not an element of {a, b, c}. It should be φ ⊂ {a, b, c}

15. Find all possible subsets of each of the following sets:

(i) A = {4, 9}

Ans. φ, {4}, {9}, {4, 9}

Note: Number of elements = 2. Total subsets =22 = 4

(ii) B = {2, 3, 8}

Ans. φ, {2}, {3}, {8}, {2, 3}, {2, 8}, {3, 8}, {2, 3, 8}

Note: Number of elements = 3. Total subsets = 23 = 8