1. Solve the following equations :

$(\mathbf{i})\ (x-4)(x+4) = (x+4)(x-7) + 33$

$(\mathbf{ii})\ (x-2)(x+3) = x^2 - 4$

$(\mathbf{iii})\ (x-1) = \frac{3}{4}(x+1) - \frac{1}{2}$

$(\mathbf{iv})\ \frac{2}{3}(x-3) = 1 - \frac{5}{6}(3x-4)$

$(\mathbf{v})\ \frac{(x-1)}{2} - \frac{(x+1)}{3} = 5 - x$

Solution:

(i) $(x-4)(x+4) = (x+4)(x-7) + 33$

$\Rightarrow x^2 - 4x + 4x - 16 = x^2 - 7x + 4x - 28 + 33$

$\Rightarrow x^2 - 16 = x^2 - 3x + 5 \Rightarrow x^2 - x^2 + 3x = 5 + 16 \Rightarrow 3x = 21$

Hence, $x = \frac{21}{3} = 7$

(ii) $(x-2)(x+3) = x^2 - 4 \Rightarrow x^2 + 3x - 2x - 6 = x^2 - 4$

$\Rightarrow x^2 + x - 6 = x^2 - 4 \Rightarrow x^2 + x - x^2 = -4 + 6$

Hence, $x = 2$

(iii) $(x-1) = \frac{3}{4}(x+1) - \frac{1}{2}$

Multiplying both sides by 4, we get

$4(x-1) = \frac{3}{4}(x+1) \times 4 - \frac{1}{2} \times 4$

$\Rightarrow 4x - 4 = 3x + 3 - 2$

$\Rightarrow 4x - 3x = 3 - 2 + 4$

Hence, $x = 5$

(iv) $\frac{2}{3}(x-3) = 1 - \frac{5}{6}(3x-4)$

Multiplying both sides by 6, we get

$6 \times \frac{2}{3}(x-3) = 1 \times 6 - 6 \times \frac{5}{6}(3x-4) \Rightarrow 4(x-3) = 6 - 5(3x-4)$

$\Rightarrow 4x - 12 = 6 - 15x + 20 \Rightarrow 4x + 15x = 6 + 20 + 12$

$\Rightarrow 19x = 38$

Hence, $x = \frac{38}{19} = 2$

(v) $\frac{x-1}{2} - \frac{x+1}{3} = 5 - x$

Multiplying both sides by 6, we get

$\Rightarrow 6 \times \frac{(x-1)}{2} - 6 \times \frac{(x+1)}{3} = 6 \times (5 - x)$

$\Rightarrow 3(x-1) - 2(x+1) = 6(5-x) \Rightarrow 3x - 3 - 2x - 2 = 30 - 6x$

$\Rightarrow 3x - 2x + 6x = 30 + 3 + 2$

$\Rightarrow 7x = 35 \Rightarrow x = \frac{35}{7} \Rightarrow x = 5$

Hence, $x = 5$

2. Find he value of a, if $x = 0.5$ is a solution of equation $ax^2 + (a-1)x + 3 = a$.

Solution:

$x = 0.5 = \frac{1}{2}$ [Given]

$x = \frac{1}{2}$ is a solution of the given equation

$ax^2 + (a-1)x + 3 = a$

Putting the value of x in the given equation, we have

$a\left(\frac{1}{2}\right)^2 + (a-1) \times \frac{1}{3} + 3 = a \Rightarrow \frac{a}{4} + \frac{a-1}{2} + 3 = a$

Multiplying both sides by 4, we get

$\Rightarrow 4 \times \frac{a}{4} + \frac{4 \times (a-1)}{2} + 3 \times 4 = 4a$

$\Rightarrow a + 2a - 2 + 12 = 4a \Rightarrow a + 2a - 4a = 2 - 12$

$\Rightarrow -a = -10$

Hence, $a = 10$

3. In a class there are x seats. If each student occupies one seat, 9 students remain standing and if two students occupy one seat, 7 seats are left empty. Find the number of seats in the class.

Solution:

Number of seats = x [Given]

Case I: When I student occupy 1 seat 9 students are left standing.

Thus number of students = x + 9

Case II: When 2 students occupy 1 seat, 7 seats are left.

Thus number of seats occupied $= x - 7$

Number of students $= 2(x - 7)$

According to the given problem,

$x + 9 = 2(x - 7) \Rightarrow x + 9 = 2x - 14 \Rightarrow x - 2x = -14 - 9$

$\Rightarrow -x = -23 \Rightarrow x = 23$

Hence, number of seats are 23.

4. The height of a triangle is 3cm more than its base. If the area of the triangle is 104 cm^2, find the lengths of its base and height.

Solution:

Let base b of a triangle (b) be x cm then height (h) be $(x + 3)$ cm and area of triangle $(A) = 104\ cm^2$

According to the given problem

$\frac{1}{2}b \times h = A \Rightarrow \frac{1}{2}x \times (x + 3) = 104$

$\Rightarrow x(x + 3) = 208 \Rightarrow x^2 + 3x - 208 = 0$

$\Rightarrow x^2 + 16x - 13x - 208 = 0 \quad \{\because 3 = 16 - 13 \text{ and } -208 = 16 \times (-13)\}$

$\Rightarrow x(x + 16) - 13(x + 16) = 0 \Rightarrow (x + 16)(x - 13) = 0$

If $x + 16 = 0$, then $x = -16$ which is not possible as it is negative

If $x - 13 = 0$ then $x = 13$

Hence, Base $= 13$ cm and height $= 13 + 3 = 16$ cm.

5. Amit walks from his house to school at a speed of 3kmph and returns back with 4 kmph. If he takes 42 min for the whole journey, find the distance between his house and the school.

Solution:

Let the distance between school and house be $x\ km$.

Total time taken $= 42$ minutes $= \frac{42}{60}$ hours

According to given problem,

$\frac{x}{3} + \frac{x}{4} = \frac{42}{60} \qquad \left[\text{since, Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

$\Rightarrow \frac{4x + 3x}{12} = \frac{42}{60} \Rightarrow \frac{7x}{12} = \frac{42}{60} \Rightarrow x = \frac{42}{60} \times \frac{12}{7} = \frac{6}{5}$

Hence, distance between his house and school is $\frac{6}{5}$ km $= 1.2$ km.

6. A boy has x coins of 50 paise each, 2x coins of 25 paise each, 4x coins of 10 paise each and 8x coins of 5 paise each. Find the number of 5 paise coins if the value of all the coins is Rs 9.

Solution:

Number of 50 paise coins be x

Number of 25 paise coins be $2x$

Number of 10 paise coins be $4x$

Number of 5 paise coins be $8x$

But, total value of coins = Rs 9

Now, according to the given problem

$x \times 50 + 2x \times 25 + 4x \times 10 + 8x \times 5 = 9 \times 100$

$\Rightarrow 50x + 50x + 40x + 40x = 900 \Rightarrow 180x = 900$

$\Rightarrow x = \frac{900}{180} = 5$

Hence, number of 5 paise coins $= 8x = 8 \times 5 = 40$

7. Solve the following :

(i) $7x + 6y = 71, 5x - 8y = -23$

(ii) $3x - y = 23, \frac{x}{3} + \frac{7}{4} = 4$

(iii) $\frac{x-3}{4} = \frac{y-7}{2}, 11x = 13y$

Solution:

(i) $7x + 6y = 71$ ... (1)

$5x - 8y = -23$ ... (2)

Multiplying equation (1) by 4 and equation (2) by 3, we get

$28x + 24y = 284$ ... (3)

$15x - 24y = -69$ ... (4)

Adding equation (3) and equation (4), we get

$43x = 215 \Rightarrow x = \frac{215}{43} = 5$

Putting $x = 5$ in equation (1), we get

$7 \times 5 + 6y = 71 \Rightarrow 35 + 6y = 71 \Rightarrow 6y = 71 - 35$

$\Rightarrow 6y = 36 \Rightarrow y = \frac{36}{6} = 6$

Hence, $x = 5$ and $y = 6$

(ii) $3x - y = 23$ ... (1)

$\frac{x}{3} + \frac{y}{4} = 4$

$\Rightarrow 12 \times \frac{x}{3} + 12 \times \frac{y}{4} = 12 \times 4$

Multiplying both sides by 12, we get

$4x + 3y = 48$

Multiplying equation (1) by 3, we get

$9x - 3y = 69$ ... (3)

$4x + 3y = 48$ ... (4)

Adding equation (3) and equation (4), we get

$9x - 3y = 69$

$4x - 3y = 48$

$\Rightarrow 13x = 117$

$\Rightarrow x = \frac{117}{13} = 9$

Putting the value of x in equation (1), we get

$3 \times 9 - y = 23 \Rightarrow 27 - y = 23 \Rightarrow -y = 23 - 27$

$\Rightarrow -y = -4 \Rightarrow y = 4$

Hence, $x = 9$ and $y = 4$

(iii) $\frac{x-3}{4} = \frac{y-7}{2}$

$2(x - 3) = 4(y - 7)$

By cross − multiplication, we get

$2x - 6 = 4y - 28$

$\Rightarrow 2x - 4y = -28 + 6 \Rightarrow 2x - 4y = -22$ ... (1)

$11x = 13y \Rightarrow 11x - 13y = 0$ ... (2)

Multiplying equation (1) by 11 and equation (2) by 2, we get

$22x - 44y = -242$ ... (3)

$22x - 26y = 0$ ... (4)

Subtraction equation (4) from (3), we get

$22x - 44y = -242$

$22x - 26y = 0$

$\Rightarrow -18y = -242$

Putting $y = \frac{121}{9}$ in equation (1), we get

$2x - 4 \times \frac{121}{9} = -22$

$\Rightarrow 2x - \frac{484}{9} = -22 \Rightarrow 2x = -22 + \frac{484}{9}$

$\Rightarrow 2x = \frac{-198 + 484}{9} \Rightarrow 2x = \frac{286}{9}$

$\Rightarrow x = \frac{289}{2 \times 9} \Rightarrow x = \frac{143}{9} \Rightarrow x = 15\frac{8}{9}$

Hence, $x = 15\frac{8}{9}$ and $y = 13\frac{4}{9}$

8. If $7x = 10y + 4$ and $12x + 18y = 1$, find the values of $(4x + 6y)$ and $(8y + x)$.

Solution:

$7x = 10y + 4$

$\Rightarrow 7x = 10\,y = 4$ ... (1)

$12x + 18y = 1$ ... (2)

Multiplying (1) by 9 and equation (2) by 5,

$63x - 90y = 36$ ... (3)

$60x + 90y = 5$ ... (4)

Adding equations (3) and (4), we get

$63x - 90y = 36$

$60x + 90y = 5$

$\Rightarrow 123x = 41$

$\Rightarrow x = \frac{41}{123} = \frac{1}{3}$

Putting the value of x in equation (1), we get

$7 \times \frac{1}{3} - 10y = 4$

$\Rightarrow \frac{7}{3} - 10y = 4 \Rightarrow -10y = 4 - \frac{7}{3} = \frac{5}{3}$

$\Rightarrow y = -\frac{5}{3 \times 10} = -\frac{1}{6}$

(i) $4x + 6y = 4 \times \frac{1}{3} + 6 \times \left(-\frac{1}{6}\right) = \frac{4}{3} - 1 = \frac{1}{3}$ and

(ii) $8y + x = 8 \times \left(-\frac{1}{6}\right) + \frac{1}{3} = \frac{-4}{3} + \frac{1}{3} = \frac{-3}{3} = -1$

9. Of the two numbers, 4 times the smaller one is less than 3 times the larger one by 6. Also, the sum of the numbers is larger than 6 times their difference by 5. Find the numbers.

Solution:

Let the larger number be x and the smaller number be y

According to the given problem

$3x - 4y = 6 \quad \ldots(1)$

$x + y - 6(x - y) = 5 \Rightarrow x + y - 6x + 6y = 5$

$\Rightarrow -5x + 7y = 5 \quad \ldots(2)$

Multiplying equation (1) by and equation (2) by 4, we get

$21x - 28y = 42 \quad \ldots(3)$

$-20x + 28y = 20 \quad \ldots(4)$

Adding equation (3) and (4), we get

$21x - 28y = 42$

$-20x + 28y = 20$

$\Rightarrow x = 62$

Putting the value of x in equation (1), we get

$3 \times 62 - 4y = 6$

$\Rightarrow 186 - 4y = 6 \Rightarrow -4y = 6 - 186 = -180 \Rightarrow y = \frac{-180}{-4} = 45$

Hence, larger number $= 62$ and smaller number $= 45$

10. Five years ago, A's age was four times the age of B. Five years hence, A's age will be twice the age of B. Find their present ages.

Solution:

Let the present age of A be x years and present age of B be y years

5 years ago, age of A $= (x - 5)$ years

5 years ago, age of B $= (y - 5)$ years

$\therefore x - 5 = 4(y - 5)$

$\Rightarrow x - 5 = 4y - 20$

$\Rightarrow x - 4y = -20 + 5$

$\Rightarrow x - 4y = -15 \quad \ldots(1)$

5 years hence, age of A $= (x + 5)$ years

5 years hence, age of B $= (y + 5)$ years

$\therefore x + 5 = 2(y + 5) \quad$ [Given]

$\Rightarrow x + 5 = 2y + 10$

$\Rightarrow x - 2y = 10 - 5$ ... (2)

$x - 2y = 5$

Subtracting equation (2) from (1), we get

$x - 4y = -15$

$x - 2y = 5$

$-2y = -20$

$\Rightarrow 2y = 20$

$\Rightarrow y = \frac{20}{2} = 10$

Putting the value of y in equation (1), we get

$x - 4 \times 10 = -15$

$\Rightarrow x = -15 + 40$

$\Rightarrow x = 25$

Hence, the percentage age of A is 25 years and B is 10 years.

11. There are two examination halls A and B. If 12 pupils are sent from A to B, the number of pupils in each halls becomes the same. If 11 pupils are sent from B to A, then the number of pupils in A is double their number in B. Find the number of pupils in each room.

Solution:

Let number of pupils in examination hall A be x and number os pupils in examination hall be y.

According to the given problem,

$x - 12 = y + 12$

$\Rightarrow x - y = 12 + 12 = 24$ ... (1)

$\Rightarrow x + 11 = 2(y - 11) \Rightarrow x + 11 = 2y - 22$

$\Rightarrow x - 2y = -22 - 11 = -33$ ... (2)

Subtracting equation (2) from equation (1), we get

$x - y = 24$

$x - 2y = -33$

$y = 57$

Putting $y = 57$ in equation (1), we get

$x - y = 24$

$x - 2y = -33$

$\Rightarrow y = 57$

Putting $y = 57$ in equation (1), we get

$\therefore x - 57 = 24$

$\Rightarrow x = 24 + 57 = 81$

Hence, number of pupils in examination hall A $= 81$.

12. A man sold a chair and a table for Rs 2178, thereby making a profit of 12% on the chair and 16% on the table. By selling them for Rs 2154, he gains 16% on the chair and 12% on the table. Find the cost price of each.

Solution:

Let CP of chair be Rs x and CP of table be Rs y.

Case I :

Gain on chair $= 12\%$

And gain on table $= 16\%$

$\therefore$ SP of chair $= \frac{x(100+12)}{100} = \frac{112}{100}x$

and SP of table $= \frac{y(100+16)}{100} = \frac{116}{100}y$

Case II :

Gain on chair $= 16\%$ and gain on table $= 12\%$

$\therefore$ SP of chair $= \frac{x(100+16)}{100} = \frac{116}{100}x$

and SP of table $= \frac{y(100+12)}{100} = \frac{112}{100}y$

According to the given problem,

$\frac{112}{100}x + \frac{116}{100}y = 2178$

$\Rightarrow 112x + 116y = 217800$ ... (1)

and $\frac{116}{100}x + \frac{112}{100}y = 2154$

$\Rightarrow 116x + 112y = 215400$ ... (2)

Adding equation (1) and equation (2), we get

$112x + 116y = 217800$

$116x + 112y = 215400$

$\Rightarrow 228x + 228y = 433200$

$\Rightarrow x + y = 1900$ ... (3)

Subtracting equation (2) from equation (1), we get

$112x + 116y = 217800$

$116x + 112y = 215400$

$-4x + 4y = 2400$

$\Rightarrow x - y = -600$ ... (4)

Adding equation (3) and equation (4), we get

$x + y = 1900$

$x - y = -600$

$\Rightarrow 2x = 1300$

$\Rightarrow x = \frac{1300}{2} = 650$

Subtracting equation (3) from (4), we get

$x - y = -600$

$x + y = 1900$

$\Rightarrow -2y = -2500$

$\Rightarrow y = \frac{-2500}{-2} = 1250$

Hence, CP of chair is Rs 650 and CP of table is Rs 1250.

13. Draw the graph of the equation $4x + 3y + 6 = 0$. From the graph, find:

(i) y_1, the value of y, when $x = 6$.

(ii) y_2, the value of y, when $x = -6$.

Solution:

$4x + 3y + 6 = 0$

$\Rightarrow 4x = -3y - 6$

$\Rightarrow x = \frac{-(3y + 6)}{4}$

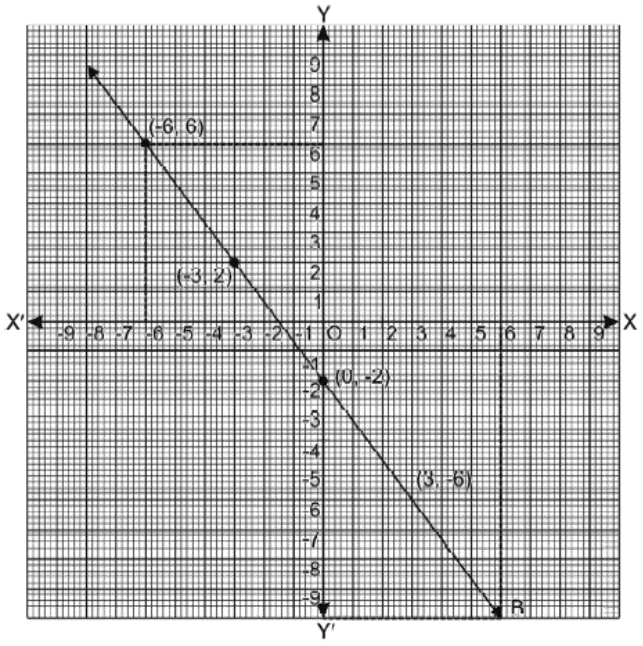
If $y = 2$, then

$x = \frac{-(3 \times 2 + 6)}{4} = \frac{-12}{4} = -3$

If $y = -2$, then

$x = \frac{-[3 \times (-2) + 6]}{4} = \frac{-[-6 + 6]}{4} = 0$

If $y = -6$, then



$$x = -\left[\frac{3 \times (-6) + 6}{4}\right] = \frac{-(-18 + 6)}{4}$$

$$= \frac{-(-12)}{4} = \frac{12}{4} = 3$$

The table of $4x + 3y + 6 = 0$

x	-3	0	3
y	2	-2	-6

Plotting the above points, we get the required graph.

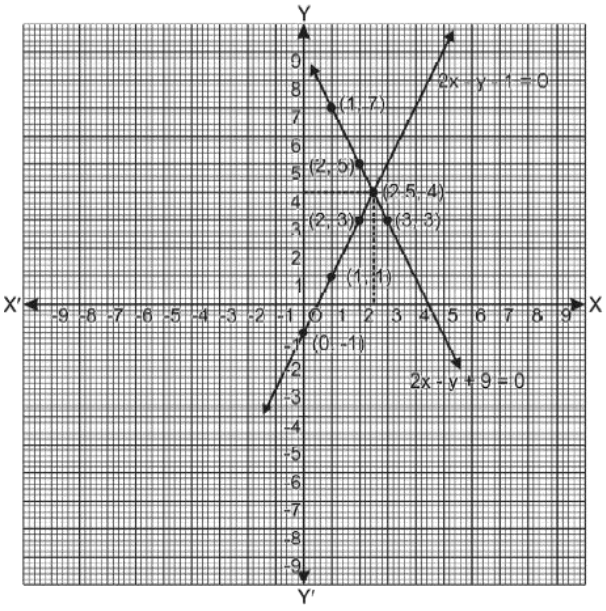
From the graph,

(i) If $x == 6$, then value of y, i. e. , y_1 will be $= -10$

(ii) If $x = -6$, then value of y, i. e. , y_2 will be $= 6$.

14. Draw the graphs of $2x - y - 1 = 0$ and $2x + y = 9$ on the same axis. Write down the co-ordinates of the point of intersection of the two lines.

Solution:



$2x - y - 1 = 0 \Rightarrow 2x = y + 1$

$$\Rightarrow x = \frac{y + 1}{2}$$

If $y = 1$, then

$$x = \frac{1 + 1}{2} = \frac{2}{2} = 1$$

If $y = 3$, then

$$x = \frac{3 + 1}{2} = \frac{4}{2} = 2$$

If $y = -1$, then

$$x = \frac{-1 + 1}{2} = 0$$

Table of $2x = y + 1$

X	1	2	0
Y	1	3	-1

Again $2x + y = 9$

$\Rightarrow y = 9 - 2x$

If $x = 1$, then $y = 9 - 2 \times 1 = 9 - 2 = 7$

If $x = 2$, then $y = 9 - 2 \times 2 = 9 - 4 = 5$

If $x = 3$, then $y = 9 - 2 \times 3 = 9 - 6 = 3$

Table of $y = 9 - 2x$

X	1	2	0
Y	1	3	-1

Plot the points given in above tables. We see that the graph of the lines nintersect each other at the point $\left(\frac{5}{2}, 4\right)$.

15. Find, graphically, the vertices of the triangle whose sides have the equations $2y - x = 8$; $5y - x = 14$ and $y - 2x = 1$ respectively. Take 1 cm = 1 unit on both the axes.

Solution:

$2y - x = 8$

$\Rightarrow x = 2y - 8$

If $y = 1$, then $x = 2 \times 1 - 8 = 2 - 8 = -6$

If $y = 3$, then $x = 2 \times 3 - 8 = 6 - 8 = -2$

If $y = 4$, then $x = 2 \times 4 - 8 = 8 - 8 = 0$

The table of $x = 2y - 8$ is

X	-6	-2	0
Y	1	3	4

Again, $5y - x = 14$

$\Rightarrow x = 5y - 14$

The table of $5y - x = 14$ is

X	-9	-4	1
Y	1	3	4

$y - 2x = 1$

$\Rightarrow y = 1 + 2x$

If $x = 2$, then

$y = 1 + 2 \times 2 = 5$

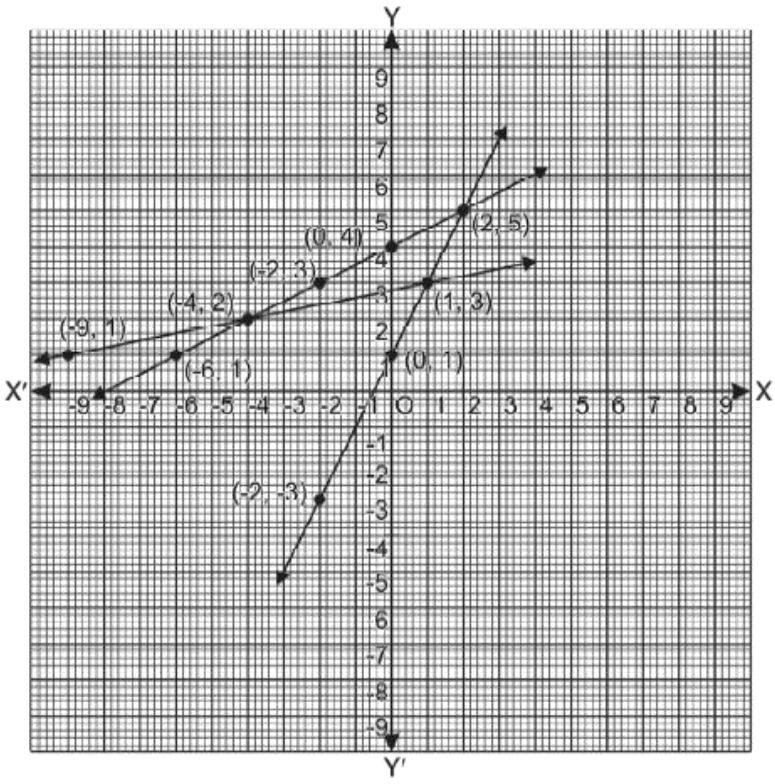
If $x = -2$, then

$y = 1 + 2 \times (-2) = -3$

If $x = 0$, then $y = 1 + 0 = 1$

The table of $y = 1 + 2x$ is

X	2	-2	0
Y	5	-3	1



Plotting the points given in the above tables, we get the required graph. From the graph the vertices of the triangle are A (−4, 2), B (2, 5) and C (1, 3).