Board – ICSE	Class – IX	Topic – Solids

1. **The length, breadth and height of a rectangular solid are in the ratio 5: 4:2. If the total surface area is 1216 cm², find the length, breadth and height of the solid.**

Solution:

$\text{Let length, } l = 5x, \text{breadth, } b = 4x \text{ and height, } h = 2x$

$\therefore \text{Total surface area} = 1216 \text{ cm}^2$

$\Rightarrow 2(lb + bh + hl) = 1216$

$\Rightarrow 2(5x \times 4x + 4x \times 2x + 2x + 5x) = 1216$

$\Rightarrow 2(20\,x^2 + 8x^2 + 10x^2) = 1216 \Rightarrow 2(38x^2) = 1216$

$\Rightarrow 76x^2 = 1216 \Rightarrow x^2 = \frac{1216}{76} = 16 \Rightarrow x^2 = 16 \Rightarrow x = 4$

$\therefore l = 5x = 5 \times 4 = 20 \text{ cm}$

$b = 4x = 4 \times 4 = 16 \text{ cm and } h = 2x = 2 \times 4 = 8 \text{ cm}$

$\text{Hence, length, } l = 20 \text{ cm, breadth, } b = 16 \text{ cm, and height } h = 8 \text{ cm.}$

2. **A class room is 12.5 m long, 6.4 m broad and 5 m high. How many students can accommodate if each student needs 1.6 m² of floor area? How many cubic metre of air would each student get?**

Solution:

$\text{Length of room } (l) = 12.5 \text{ m, width of room } (b) = 6.4 \text{ m and height of room } (h) = 5 \text{ m}$

$\therefore \text{Volume of air inside the room} = l \times b \times h = 12.5 \times 6.4 \times 5 \text{ m}^3 = 400 \text{ m}^3$

$\text{Area of floor of the room} = l \times b = 12.5 \times 6.4 \text{ m}^2 = 80 \text{ m}^2$

$\text{For each student area required} = 1.6 \text{ m}^2$

$\therefore \text{Number of students} = \frac{80}{1.6} = \frac{80 \times 10}{16} = 50$

$\text{The required air received by each student} = \frac{\text{Volume of air}}{\text{Number of students}} = \frac{400}{50} = 8 \text{ m}^3$

3. **The volume of a cuboid is 14400 3 cm and its height is 15 cm. The cross-section of the cuboid is a rectangle having its sides in the ratio 5: 3. Find the perimeter of the cross-section.**

Solution:

$\text{Volume of cuboid} = 14400 \text{ cm}^3, \text{and height of cuboid } (h) = 15 \text{ cm}$

$\therefore \text{Length} \times \text{Breadth} = \frac{\text{volume}}{h} = \frac{14400}{15} \text{ cm}^2 = 960 \text{ cm}^2$

Ratio of remaining sides $= 5 : 3$

Thus, length $= 5$ x and breadth $= 3x$

$\therefore 5x \times 3x = 960 \Rightarrow 15x^2 = 960 \Rightarrow x^2 = 64 = (8)^2$

$\therefore x = 8$

Length $= 5x = 8 \times 5 = 40$ cm and breadth $= 3x = 8 \times 3 = 24$ cm

Hence, perimeter of rectangular cross $-$ section $= 2(l + b) = 2(40 + 24)$ cm

$= 2 \times 64 = 128$ cm.

4. The cost of papering the four walls of a room 12 m long at Rs 6.50 per square metre is Rs 1638 and the cost of matting the floor at Rs 3.50 per square metre is Rs 378. Find the height of the room.

Solution:

Rate of papering the walls $=$ Rs 6.50 per m^2 and total cost $=$ Rs 1638

$\therefore$ Area of four walls $= \frac{1638}{6.50} = \frac{1638 \times 100}{650} m^2 = 252\ m^2$

Rate of matting the floor $=$ Rs 3.50 per m^2 and total cost $=$ Rs 378

$\therefore$ Area of floor $= \frac{378}{3.50} = \frac{378 \times 100}{350} m^2 = 108\ m^2$

Length of room $= 12$ m

$\therefore$ Breadth of room $= \frac{\text{Area of floor}}{\text{Length}} = \frac{108}{2} = 9$ m

Area of four walls $= 2(l + b)h = 252$

$\Rightarrow 2(12 + 9)h = 252 \Rightarrow 2 \times 2lh = 252$

$\Rightarrow h = \frac{252}{2 \times 21} \Rightarrow h = 6$

Hence, height of the room is 6 m.

5. The sum of length, breadth and depth of a cuboid is 19 cm, and the length of its diagonal is 11 cm. Find the surface area of the cuboid.

Solution:

Let l, b and h be the length, breadth and depth of the cuboid, then

$l + b + h = 19$ cm

and $\sqrt{l^2 + b^2 + h^2} = 11$ cm $\Rightarrow l^2 + b^2 + h^2 = (11)^2 = 121$

The surface area of the cuboid $= 2(lb + bh + hl)$

We know that $(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

$\Rightarrow (19)^2 = 121 + 2(lb + bh + hl) \Rightarrow 2(lb + bh + hl) = (19)^2 - 121$

$= 361 - 121 = 240\ cm^2$

Hence, surface area of the cuboid $= 240\ cm^2$

6. **The total surface area of a cube is 726 cm^2. Find its volume.**

Solution:

Total surface area of cube $= 726\ cm^2$

Let edge of cube $=$ a cm

$\therefore$ Total surface area of cube $= 6a^2 = 726$

$\Rightarrow a^2 = \frac{726}{6} = 121 \Rightarrow a = \sqrt{121} = 11$ cm

$\therefore$ Edge of cube $= 11$ cm

Volume of cube $= (side)^3 = (11\ cm)^3$

$= 11 \times 11 \times 11\ cm^3$

$= 1331\ cm^3$

7. **The edges of three cubes of metal are 3 cm, 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.**

Solution:

The edges of three cubes are 3 cm, 4 cm and 5 cm.

Volume of three cubes are $(3)^3$, $(4)^3$ and $(5)^3$ i.e., 27 cm^3, 64 cm^3 and 125 cm^3

$\therefore$ Volume of new cube $= 27 + 64 + 125 = 216\ cm^3$

Let edge of new cube be a

8. **Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.**

Solution:

Let edge of each of three cubes be 'x' when three cubes are placed to end, a cuboid is formed whose dimensions are :

$l = x + x + x = 3x, b = x, h = x$

$\therefore$ Total surface area of a resulting cuboid $= 2(lb + bh + hl)$

$= 2(3x \times x + x \times x + x \times 3x)$

$= 2(3x^2 + x^2 + 3x^2) = 2 \times 7x^2 = 14x^2$

Sum of total surface areas of three cubes $= 3 \times 6x^2 = 18x^2$

$\therefore$ Ratio of surface areas of cubes $= \frac{14x^2}{18x^2} = \frac{14}{18} = \frac{7}{9}$

Hence, the required ratio is $7 : 9$

9. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find

(i) The edge of the cube

(ii) The surface area of the cube.

Solution:

Length of the metallic cuboid $(l) = 100$ cm

Breadth of the metallic cuboid $(b) = 80$ cm

And height of metallic cuboid $(h) = 64$ cm

$\therefore$ Volume of metallic cuboid $= l \times b \times = 100 \times 80 \times 64 = 512000\ cm^3$

$\therefore$ Volume of cube so formed $= 512000\ cm^3$

(i)Edge of the cube $(a) = (512000)^{1/3} = [(80)^3]^{1/3} = 80$ cm

(ii) Surface area of cube $= 6a^2 = 6(80)^2 = 6 \times 80 \times 80\ cm^2 = 38400\ cm^2$

10. Four identical cubes are joined end to end to form a cuboid. If the total surface area of the resulting cuboid is 648 cm^2. Find the length of edge of each cube. Also find the ratio between the surface area of resulting cuboid and the surface area of a cube.

Solution:

Surface area of cuboid $= 648\ cm^2$, let edge of each cube $= x$ cm

$\therefore$ Length of cuboid $= 4x$ cm, width of cuboid $= x$ cm and height of cuboid $= x$ cm

$\therefore$ Surface area of cuboid $= 2(lb + bh + hl)$

$\Rightarrow 648 = 2(4x \times x + x \times x + x \times 4x) = 2(4x^2 + x^2 + 4x^2)$

$\Rightarrow 648 = 18x^2 \Rightarrow x^2 = \frac{648}{18} = 36 = (6)^2$

$\therefore x = 6$

Thus, length of edge of each cube $= 6$ cm

Surface area of cube $= 6(\text{side})^2 = 6x^2 = 6 \times (6)^2 = 6 \times 36 = 216\ cm^2$

Hence, ratio between the surface area of cuboid and that of cube $= 648 : 216 = 3 : 1$

11. A rectangular container whose base is a square of side 12 cm, contains sufficient water to submerge a rectangular solid 8 cm × 6 cm × 3 cm. Find the rise in level of water in the container when the solid is in it.

Solution:

The rise in water level = x cm

Length of solid cuboid = 8 cm,

Width of solid cuboid = 6 cm

Height of solid cuboid = 3 cm

$\therefore$ Volume of cuboid $= l \times b \times h = 8 \times 6 \times 3 = 144\ cm^3$

$\therefore$ Volume of water $= 144\ cm^3$

Side of the square base = 12 cm

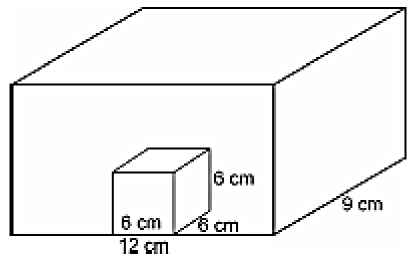
$\therefore$ Area of base $= 12 \times 12 = 144\ cm^2$

According to given problems, we get

$144 \times x = 144$

$\Rightarrow x = 1$

Hence, rise in water level is 1 cm.



12. The following figure shows a solid of uniform cross-section. Find the volume of the solid. All measurements are in centimeters. Assume that all angles in the figure are right angles.

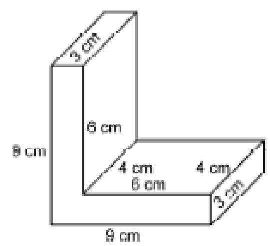
Solution:

The given figure can be divided into two cuboids of dimensions

6 cm, 4 cm, 3 cm and 9 cm

Respectively. Hence, volume of solid = 6× 4×3 + 4×3×9 3

= 72 +108 = 180 cm^3

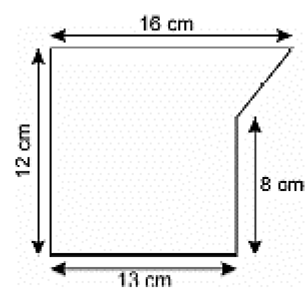


13. The cross-section of a piece of metal 2 m in length is shown in the adjoining figure. Calculate:

(i) The area of its cross-section.

(ii) The volume of piece of metal.

(iii) The weight of piece of metal to the nearest kg, if 1 cm^3 of the metal weighs 6.5 g.



Solution:

From C, draw CF||AB

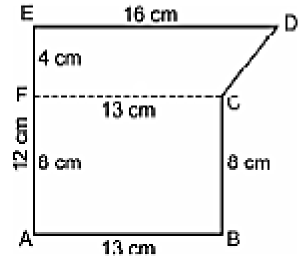
Then CF = AB = 13 cm

AF = CB = 8 cm

$\therefore$ EF = EA – FA = 12 – 8 = 4 cm

(i) Now are of figure ABCDE (cross – section)

$= \text{ar. (Rectangle ABCF)} + \text{ar. (Trapezium FCDE)}$



$= 13\text{ cm} \times 8\text{ cm} + \frac{1}{2}(13 + 16) \times 4\text{ cm}^2$

$= 104 + 29 \times 2 = 104 + 58$

$= 162\text{ cm}^2$

(ii) Length of the piece = 2 m = 200 cm

$\therefore$ Volume of the metal piece = Area × Length

$= 162 \times 200\text{ cm}^3$

$= 32400\text{ cm}^3$

$= \frac{32400}{100 \times 100} = 3.24\text{ m}^3$

(iii) Weight of 1 cm^3 metal = 6.5 g

$\therefore$ Total weight of the metal piece = 32400 × 6.5 g

$= 210600\text{ g}$

$= 211\text{ kg (approx.)}$

14. The area of cross-section of a rectangular pipe is 2 5.4 cm and water is pumped out of it at the rate of 27 kmph. Find, in liters, the volume of water which flows out of the pipe in 1 minute.

Solution:

Area of cross – section of rectangular pipe = 5.4 cm^2

Speed of water pumped out = 27 kmph

Time = 1 minute

$\therefore$ Length of water flow $= \frac{27}{60} \times 1000\text{ m} = 450\text{ m}$

$\therefore$ Volume of water = Area × Length

$= 450 \times \frac{5.4}{100 \times 100}\text{m}^3$

$= \frac{450 \times 54}{100 \times 100 \times 10}\text{m}^3$

$= \frac{24300}{100000}\text{m}^3$

$= \frac{243}{1000}\text{m}^3$

$\therefore$ Volume of water in litres $= \frac{243}{1000} \times 1000$ $\quad (1\ \text{m}^3 = 1000\ \text{litres})$

$= 243$ litres

15. A stream, which flows at a uniform rate of 4 km/hr, is 10 meters wide and 1.2 m deep at a certain point. If its cross-section is rectangular is shape find, in liters, the volume of water that flows in a minute.

Solution:

Speed of stream $= 4\frac{\text{km}}{\text{hr}} = 4 \times \frac{\frac{5}{18}\text{m}}{\text{s}} = \frac{\frac{10}{9}\text{m}}{\text{s}}$

Area of cross $-$ section of stream $= 10 \times 1.2 = 12\ \text{m}^2$

$\therefore$ Volume of water discharged in 1 second $= 12 \times \frac{10}{9} = \frac{40}{3}\text{m}^3$

But $1\ \text{m}^3 = 1000$ litres

$\therefore \frac{40}{3}\text{m}^3 = \frac{40}{3} \times 1000 = \frac{40000}{3}$ litres

$\therefore$ Volume of water discharged in 1 minutes i. e. 60 sec. $= \frac{40000}{3} \times 60 = 800000$ litres