Class – 9th	Topic – Trigonometry

1. From the following figure find the values of :

(i) sin B

(ii) tan C

(iii) $\sec^2 B - \tan^2 B$

(iv) $\sin^2 C + \cos^2 C$

Solution:

In right angled ΔABD,

$\Rightarrow AB^2 = BD^2 + AD^2$ (Using Pythagoras Theorem)

$\Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow AD = \sqrt{AB^2 - BD^2} = \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25} = \sqrt{144} = 12$

In right angled ΔACD,

$AC^2 = AD^2 + DC^2$

$\Rightarrow AC = \sqrt{AD^2 + DC^2} = \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256} = \sqrt{400} = 200$

(i) $\sin B = \frac{AD}{AB} = \frac{12}{13}$

(ii) $\tan C = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$

(iii) $\sec^2 B - \tan^2 B, \sec B = \frac{AB}{BD} = \frac{13}{5}$

$\tan B = \frac{AD}{BD} = \frac{12}{5}$

$\therefore \sec^2 B - \tan^2 B = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 = \frac{169}{25} - \frac{144}{25} = \frac{169 - 144}{2} = \frac{25}{25} = 1$

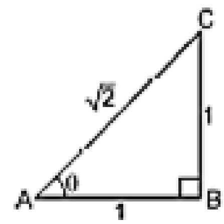
(iv) $\sin^2 C + \cos^2 C$

$\sin C = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$

$\cos C = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$

$\therefore \sin^2 C + \cos^2 C = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$

2. If $\tan\theta = \frac{8}{15}$, find the values of other trigonometrical ratios for θ



Solution:

$\sin\theta = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$

$\therefore BC = 1$ and $AC = \sqrt{2}$

In right $\Delta ABC, \angle B = 90°$

$AC^2 = AB^2 + BC^2$ (Using Pythagoras Theorem)

$\Rightarrow (\sqrt{2})^2 = (AB)^2 + (1)^2 \Rightarrow 2 = AB^2 + 1$

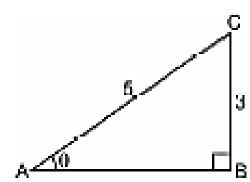
$\Rightarrow AB^2 = 2 - 1 = 1 = (1)^2$

$\therefore AB = 1$

Now, $\cos\theta = \frac{AB}{AC} = \frac{1}{\sqrt{2}}, \tan\theta = \frac{1}{1} = 1, \cot\theta = \frac{1}{1} = 1$

$\sec\theta = \frac{\sqrt{2}}{1} = \sqrt{2}, \operatorname{cosec}\theta = \frac{\sqrt{2}}{1} = \sqrt{2}$

3. If $\sin\theta = \frac{3}{5}$ and θ is an acute angle, find the values of $\cos\theta$ and $\tan\theta$.



Solution:

$\sin\theta = \frac{3}{5} = \frac{BC}{AC} \therefore BC = 3$ and $AC = 5$

In $\Delta ABC, AC^2 = AB^2 + BC^2$

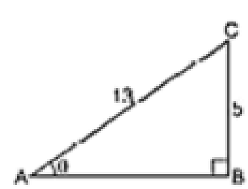
$\Rightarrow (5)^2 = AB^2 + (3)^2 \Rightarrow 25 = AB^2 + 9$

$\Rightarrow AB^2 = 25 - 9 = 16 = (4)^2$

$\therefore AB = 4$

$\cos\theta = \frac{AB}{AC} = \frac{4}{5}, \tan\theta = \frac{BC}{AB} = \frac{3}{4}$

4. If $13\sin\theta = 5$, find the value of $\frac{(5\sin\theta - 2\cos\theta)}{\tan\theta}$



Solution:

$13\sin\theta = 5 \Rightarrow \sin\theta = \frac{5}{13}$. But $\sin\theta = \frac{BC}{AC} = \frac{5}{13}$

$\therefore BC = 5, AC = 13$

Now in ΔABC,

$AC^2 = AB^2 + BC^2$ (Using Pythagoras Theorem)

$\Rightarrow (13)^2 = AB^2 + (5)^2 \Rightarrow 169 = AB^2 + 25$

$\Rightarrow AB^2 = 169 - 25 = 144 \Rightarrow AB^2 = (12)^2$

$\therefore AB = 12$

Now, $\cos\theta = \frac{AB}{AC} = \frac{12}{13}, \tan\theta = \frac{BC}{AB} = \frac{5}{12}$

$$\therefore \frac{5\sin\theta - 2\cos\theta}{\tan\theta} = \frac{5 \times \frac{5}{13} - 2 \times \frac{12}{13}}{\frac{5}{12}} = \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}} = \frac{\frac{1}{13}}{\frac{5}{12}} = \frac{1}{13} \times \frac{12}{5} = \frac{12}{65}$$

5. **If $\sec\theta = \frac{13}{5}$, show that $\left(\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}\right) = 3$**

Solution:

$\sec\theta = \frac{13}{5} = \frac{AC}{AB} \therefore AC = 13, AB = 5$

Now, in right ΔABC,

$AC^2 = AB^2 + BC^2 \Rightarrow (13)^2 = (5)^2 + (BC)^2$

$\Rightarrow 169 = 25 + BC^2 \Rightarrow BC^2 = 169 - 25 = 144 = (12)^2$

$\therefore BC = 12$

Now, $\sin\theta = \frac{BC}{AC} = \frac{12}{13}, \cos\theta = \frac{AB}{AC} = \frac{5}{13}$

Now, LHS $= \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$$= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}} = \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13} \times \frac{13}{3} = 3 = \text{RHS}$$

6. **If $3\tan\theta = 4$, show that $\left(\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}\right) = 3$**

Solution:

$3\tan\theta = 4 \therefore \tan\theta = \frac{4}{3}$

Now, LHS $= \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3\frac{\sin\theta}{\cos\theta} + 2\frac{\cos\theta}{\cos\theta}}{3\frac{\sin\theta}{\cos\theta} - 2\frac{\cos\theta}{\cos\theta}} = \frac{3\tan\theta + 2}{2\tan\theta - 2}$

Dividing numerator and denominator by $\cos\theta$

Putting the value of $3\tan\theta = 4$

$$= \frac{4+2}{4-2} = \frac{6}{2} = 3 = \text{RHS}$$

7. If $\cot\theta = \frac{q}{p}$, show that $\left(\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\sin\theta}\right) = \left(\frac{p^2-q^2}{p^2+q^2}\right)$

Solution:

$$\cot\theta = \frac{q}{p}$$

$$\text{LHS} = \frac{p\sin\theta - q\sin\theta}{p\sin\theta + q\cos\theta}$$

$$= \frac{p\frac{\sin\theta}{\sin\theta} - q\frac{\cos\theta}{\sin\theta}}{p\frac{\sin\theta}{\sin\theta} + q\frac{\cos\theta}{\sin\theta}}$$ (Dividing numerator and denominator by $\sin\theta$)

$$= \frac{p - q\cot\theta}{p + q\cot\theta} = \frac{p - q\times\frac{q}{p}}{p + q\times\frac{q}{p}} = \frac{p - \frac{q^2}{p}}{p + \frac{q^2}{p}} = \frac{p^2-q^2}{p} \times \frac{p}{p^2+q^2} = \frac{p^2-q^2}{p^2+q^2} = \text{RHS}$$

8. Use the adjoining figure and write the values of :

(i) $\sin x°$

(ii) $\cos y°$

(iii) $3\tan x° - 2\sin y° + 4\cos y°$

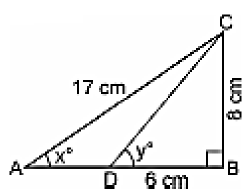
Solution:

In $\Delta ABC, \angle B = 90°$

$AC^2 = AB^2 + BC^2 \Rightarrow (17)^2 = AB^2 + (8)^2$

$\Rightarrow 289 = AB^2 + 64 \Rightarrow AB^2 = 289 - 64 = 225 = (15)^2$

$\therefore AB = 15$ cm



and in $\Delta BCD, \angle B = 90°$

$CD^2 = BD^2 + BC^2$ (Using Pythagoras Theorem)

$= (6)^2 + (8)^2 = 36 + 64 = 100 = (10)^2$

$\therefore CD = 10$ cm

(i) $\sin x° = \frac{BC}{AC} = \frac{8}{17}$

(ii) $\cos y° = \frac{BD}{CD} = \frac{6}{10}$

(iii) $\tan x° = \frac{BC}{AB} = \frac{8}{15}, \sin y° = \frac{BC}{CD} = \frac{8}{10} = \frac{4}{5}$

$\therefore 3\tan x° - 2\sin y° + 4\cos y° = 3 \times \frac{8}{15} - 2 \times \frac{4}{5} + 4 \times \frac{3}{5} = \frac{8}{5} - \frac{8}{5} + \frac{12}{5} = \frac{12}{5} = 2\frac{2}{5}$

9. If $(\cos\theta + \sec\theta = \frac{5}{2}$, find the value of $(\cos^2\theta + \sec^2\theta)$

Solution:

$(\cos\theta + \sec\theta) = \frac{5}{2}$

Squaring both sides, we get

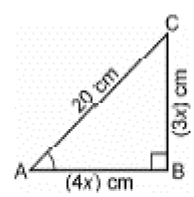
$(\cos\theta + \sec\theta) = \left(\frac{5}{2}\right)^2$

$\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta = \frac{25}{4}$ $\quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \cos^2\theta + \sec^2\theta + 2\cos\theta \times \frac{1}{\cos\theta} = \frac{25}{4}$

$\Rightarrow \cos^2\theta + \sec^2\theta + 2 = \frac{25}{4} \Rightarrow \cos^2\theta + \sec^2\theta = \frac{17}{4}$

10. In the given figure, ΔABC is right angled at B. If AC $= 20$ cm and $\tan A = \frac{3}{4}$, find the length of AB and BC



Solution:

In $\Delta ABC, \angle B = 90°$

$AC = 20$ cm, $\tan A = \frac{3}{4}$

But $\tan A = \frac{BC}{AB} = \frac{3}{4} = \frac{3x}{4x}$ $\therefore BC = 3x$ cm and $AB = 4x$ cm

$AC^2 = AB^2 + BC^2 \Rightarrow (20)^2 = (4x)^2 + (3x)^2$

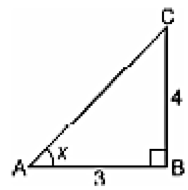
$\Rightarrow 400 = 16x^2 + 9x^2 = 25x^2$

$\therefore x^2 = \frac{400}{25} = 16 = (4)^2$

$\therefore x = 4$

Hence, $AB = 4x = 4 \times 4 = 16$ cm and $BC = 3x = 3 \times 4 = 12$ cm

11. If $\tan x = 1\frac{1}{3}$ find the value of $4\sin^2 x - 3\cos^2 x + 2$



Solution:

$$\tan x = 1\frac{1}{3} = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$\therefore BC = 4, AB = 3$

$AC^2 = AB^2 + BC^2$ (Using Pythagoras Theorem)

$= (3)^2 + (4)^2 = 9 + 16 = 25 = (5)^2$

$\therefore AC = 5$

$$\therefore \sin x = \frac{BC}{AC} = \frac{4}{5} \text{ and } \cos x = \frac{AB}{AC} = \frac{3}{5}$$

$$\therefore 4\sin^2 x - 3\cos^2 x + 2 = 4\left(\frac{4}{5}\right)^2 - 3\left(\frac{3}{5}\right)^2 + 2$$

$$= 4 \times \frac{16}{25} - 3 \times \frac{9}{25} + 2$$

$$= \frac{64}{25} - \frac{27}{25} + 2$$

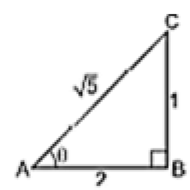
$$= \frac{64 - 27 + 50}{2} = \frac{87}{25} = 3\frac{12}{25}$$

12. If $\text{cosec}\theta = \sqrt{5}$, find the value of:

(i) $2 - \sin^2\theta - \cos^2\theta$

(ii) $2 + \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$

Solution:



$$\text{cosec}\,\theta = \frac{\sqrt{5}}{1} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}$$

$\therefore AC = \sqrt{5}, BC = 1$

But $AC^2 = AB^2 + BC^2$ (Using Pythagoras Theorem)

$\Rightarrow \left(\sqrt{5}\right)^2 = AB^2 + (1)^2 \Rightarrow 5 = AB^2 + 1$

$\Rightarrow AB^2 = 5 - 1 = 4 = (2)^2$

$\therefore AB = 2$

$$\text{Now } \sin\theta = \frac{BC}{AC} = \frac{1}{\sqrt{5}}, \cos\theta = \frac{AB}{AC} = \frac{2}{\sqrt{5}}$$

(i) $2 - \sin^2\theta - \cos^2\theta = 2 - \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2 = 2 - \frac{1}{5} - \frac{4}{5} = \frac{10-1-4}{5} = \frac{5}{5} = 1$

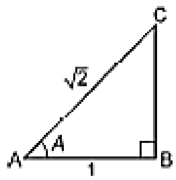
(ii) $2 + \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} = 2 + \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} - \frac{\left(\frac{2}{\sqrt{5}}\right)^2}{\left(\frac{1}{\sqrt{5}}\right)^2} = 2 + \frac{1}{\frac{1}{5}} - \frac{\frac{4}{5}}{\frac{1}{5}} = 2 + \frac{5}{1} - \frac{4}{5} \times \frac{5}{1}$

$= 2 + 5 - 4 = 7 - 4 = 3$

13. If $\sec A = \sqrt{2}$, find the value of $\frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A}$

Solution:

$\sec A = \sqrt{2} = \frac{\sqrt{2}}{1}$



$\frac{\text{Hyp.}}{\text{Base}} = \frac{AC}{AB} = \frac{\sqrt{2}}{1}$

$\therefore \quad AC = \sqrt{2},\ AB = 1$

But $AC^2 = AB^2 + BC^2$ (Using Pythagoras Theorem)

$\Rightarrow (\sqrt{2})^2 = (1)^2 + (BC)^2 \Rightarrow 2 = 1 + BC^2$

$\Rightarrow BC^2 = 2 - 1 = (1)^2 \ \therefore\ BC = 1$

$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{2}},\quad \sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}},\quad \tan A = \frac{BC}{AB} = \frac{1}{1} = 1$

$\therefore \quad \frac{3\cos^2 A + 5\tan^2 A}{4\tan^2 A - \sin^2 A} = \frac{3\left(\frac{1}{\sqrt{2}}\right)^2 + 5 \times (1)^2}{4 \times (1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{3 \times \frac{1}{2} + 5 \times 1}{4 \times 1 - \frac{1}{2}} = \frac{\frac{3}{2} + 5}{4 - \frac{1}{2}} = \frac{\frac{13}{2}}{\frac{7}{2}} = \frac{13}{2} \times \frac{2}{7} = \frac{13}{7} = 1\frac{6}{7}.$

14. In the following figure, $BD = 18, DC = 24, AC = 50, \angle A = x, \angle BCD = y$ and $\angle ABC = 90° = \angle BDC$ find:

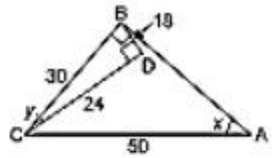
a) $2\tan x - \sin y$

b) $3 - 2\cos x + 3\cot y$

c) $5 - 3\tan^2 x + 3\sec^2 x$

d) $2\cot^2 y - 2\mathrm{cosec}^2 y + 3$

Solution:



$$BC^2 = BD^2 + CD^2$$
$$= (18)^2 + (24)^2 = 324 + 576 = 900 = (30)^2$$
$$\therefore \quad BC = 30$$

In ΔABC, $\angle B = 90°$

$$AC^2 = AB^2 + BC^2 \Rightarrow (50)^2 = AB^2 + (30)^2$$
$$\Rightarrow \quad 2500 = AB^2 + 900 \Rightarrow AB^2 = 2500 - 900 = 1600 = (40)^2$$
$$\therefore \quad AB = 40$$

$$\sin x = \frac{BC}{AC} = \frac{30}{50} = \frac{3}{5}, \ \cos x = \frac{AB}{AC} = \frac{40}{50} = \frac{4}{5}$$
$$\tan x = \frac{BC}{AB} = \frac{30}{40} = \frac{3}{4}, \ \sec x = \frac{1}{\cos x} = \frac{5}{4}$$
$$\sin y = \frac{BD}{BC} = \frac{18}{30} = \frac{3}{5}, \ \cot y = \frac{CD}{BD} = \frac{24}{18} = \frac{4}{3}$$

(i) $2\tan x - \sin y = 2\times\frac{3}{4} - \frac{3}{5} = \frac{3}{2} - \frac{3}{5} = \frac{15-6}{10} = \frac{9}{10}$

(ii) $3 - 2\cos x + 3\cot y = 3 - 2\times\frac{4}{5} + 3\times\frac{4}{3} = 3 - \frac{8}{5} + 4 = 7 - \frac{8}{5} = \frac{35-8}{5} = \frac{27}{5} = 5\frac{2}{3}$

(iii) $5 - 3\tan^2 x + 3\sec^2 x = 5 - 3\times\left(\frac{3}{4}\right)^2 + 3\left(\frac{5}{4}\right)^2 = 5 - 3\times\frac{9}{16} + 3\times\frac{25}{16}$

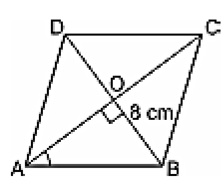
$$= 5 - \frac{27}{16} + \frac{75}{16} = \frac{80-27+75}{16} = \frac{155-27}{16} = \frac{128}{16} = 8$$

(iv) $2\cot^2 y - 2\operatorname{cosec}^2 y + 3 = 2\left(\frac{4}{3}\right)^2 - 2\left(\frac{5}{3}\right)^2 + 3$

$$= 2\times\frac{16}{9} - 2\times\frac{25}{9} + 3 = \frac{32}{9} - \frac{50}{9} + 3$$
$$= \frac{32-50+27}{9} = \frac{59-50}{9} = \frac{9}{9} = 1.$$

15. In rhombus ABCD, diagonals AC and BD intersect each other at point O. If cosine of angle CAB is 0.6 and OB = 8 cm, find the length of the side and the diagonals of the rhombus.

Solution:



$$\cos\angle CAB = 0.6$$
$$= \frac{6}{10} = \frac{3}{5} = \frac{OA}{AB}$$

Let $OA = 3x$, $AB = 5x$

In ΔAOB,

$$AB^2 = AO^2 + OB^2 \Rightarrow (5x)^2 = (3x)^2 + OB^2$$
$$\Rightarrow \quad 25x^2 = 9x^2 + OB^2$$
$$\Rightarrow \quad OB^2 = 25x^2 - 9x^2 = 16x^2 = (4x)^2 \ \therefore \ OB = 4x$$

But, $OB = 8 \text{ cm} \Rightarrow 4x = 8 \text{ cm} \Rightarrow x = \frac{8}{4} \text{ cm} \ \therefore \ OA = \frac{8}{4}\times 3 = 6 \text{ cm}$

and $AB = \frac{8}{4}\times 5 = 10 \text{ cm}$

Hence, each side $= 10$ cm and diagonal $AC = 2OA = 2\times 6 = 12$ cm and diagonal $BD = 2OB = 2\times 8 = 16$ cm.

16. If $\sin A = \cos A$, find the value of $2\tan^2 A - 2\sec^2 A + 5$

Solution:

$$\sin A = \cos A \Rightarrow \frac{\sin A}{\cos A} = 1 \Rightarrow \tan A = 1$$

$$\therefore\ 2\tan^2 A - 2\sec^2 A + 5 = 2\tan^2 A - 2(1+\tan^2 A) + 5 \quad (\because \sec^2 A = 1 + \tan^2 A)$$

$$= 2\tan^2 A - 2 - 2\tan^2 A + 5 = -2 + 5 = 3.$$

17. If $2\sin x = \sqrt{3}$, Evaluate:

a) $4\sin^3 x - 3\sin x$

b) $3\cos x - 4\cos^3 x$

Solution:

$$2\sin x = \sqrt{3}$$

$$\Rightarrow\ \sin x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

(i) $4\sin^3 x - 3\sin x = \sin x\,(4\sin^2 x - 3)$

$$= \frac{\sqrt{3}}{2}\left[4\left(\frac{\sqrt{3}}{2}\right)^2 - 3\right] = \frac{\sqrt{3}}{2}\left(4\times\frac{3}{4} - 3\right) = \frac{\sqrt{3}}{2}(3-3) = \frac{\sqrt{3}}{2}\times 0 = 0.$$

(ii) $3\cos x - 4\cos^3 x = \cos x\,(3 - 4\cos^2 x)$

$$= \frac{1}{2}\left\{3 - 4\times\left(\frac{1}{2}\right)^2\right\} = \frac{1}{2}\left(3 - 4\times\frac{1}{4}\right) = \frac{1}{2}(3-1) = \frac{1}{2}\times 2 = 1.$$

18. If $2\sin x = \sqrt{3}$, evaluate:

a) $4\sin^3 x - 3\sin x$

b) $3\cos x - 4\cos^3 x$

Solution:

$$2\sin x = \sqrt{3}$$

$$\Rightarrow\ \sin x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{4-3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

(i) $4\sin^3 x - 3\sin x = \sin x\,(4\sin^2 x - 3)$

$$= \frac{\sqrt{3}}{2}\left[4\left(\frac{\sqrt{3}}{2}\right)^2 - 3\right] = \frac{\sqrt{3}}{2}\left(4\times\frac{3}{4} - 3\right) = \frac{\sqrt{3}}{2}(3-3) = \frac{\sqrt{3}}{2}\times 0 = 0.$$

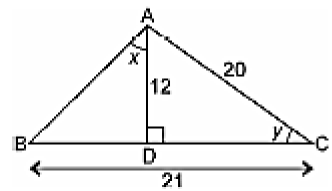
(ii) $3\cos x - 4\cos^3 x = \cos x\,(3 - 4\cos^2 x)$

$$= \frac{1}{2}\left\{3 - 4\times\left(\frac{1}{2}\right)^2\right\} = \frac{1}{2}\left(3 - 4\times\frac{1}{4}\right) = \frac{1}{2}(3-1) = \frac{1}{2}\times 2 = 1.$$

19. Use the information given in the following figure to evaluate :

$$\frac{10}{\sin x}+\frac{6}{\sin y}-6\cot y$$

Solution:



In ΔADC, we get

$AC^2 = AD^2 + DC^2$ (Using Pythagoras Theorem)

$\Rightarrow 20^2 = 12^2 + DC^2$

$\Rightarrow 400 = 144 + DC^2$

$\Rightarrow DC^2 = 400 - 144 = 256 = (16)^2$

$\therefore DC = 16$

But $BC = 21$

$\therefore BD = BC - DC = 21 - 16 = 5$

In ΔADB,

$AB^2 = AD^2 + BD^2$

$= (12)^2 + (5)^2 = 144 + 25 = 169 = (13)^2$

$\therefore AB = 13$

$\sin x = \frac{BD}{AB} = \frac{5}{13}$, $\sin y = \frac{AD}{AC} = \frac{12}{20} = \frac{3}{5}$, $\cot y = \frac{CD}{AD} = \frac{16}{12} = \frac{4}{3}$

Now $\frac{10}{\sin x} + \frac{6}{\sin y} - 6\cot y = \frac{10}{\frac{5}{13}} + \frac{6}{\frac{3}{5}} - 6\left(\frac{4}{3}\right) = \frac{10\times 13}{5} + \frac{6\times 5}{3} - 6\times\frac{4}{3}$

$= 26 + 10 - 8 = 36 - 8 = 28.$