SpeedLabs

MATHS

ICSE 8th

TEEVRA EDUTECH PVT. LTD.

Special Quadrilateral

1. Prove that any two adjacent angles of a parallelogram are supplementary.

Ans. Let ABCD be a parallelogram

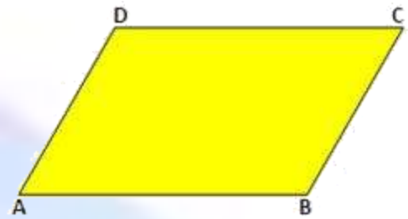
Then, AD || BC and AB is a transversal.

Therefore, A + B = 180°[Since, sum of the interior angles on the same side of the transversal is 180°]

Similarly, ∠B + ∠C = 180°,

∠C + ∠D = 180° and

∠D + ∠A = 180°.



Thus, the sum of any two adjacent angles of a parallelogram is 180°.

Hence, any two adjacent angles of a parallelogram are supplementary.

2. Two adjacent angles of a parallelogram are as 2 : 3. Find the measure of each of its angles

Ans. Let ABCD be a given parallelogram

Then, ∠A and ∠B are its adjacent angles.

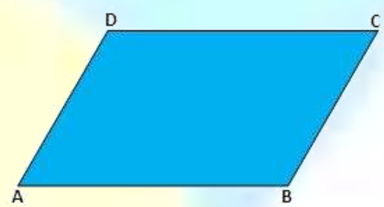
Let ∠A = (2x)° and ∠B = (3x)°.

Then, ∠A + ∠B = 180° [Since, sum of adjacent angles of a || gm is 180°]

⇒ 2x + 3x = 180

⇒ 5x = 180

⇒ x = 36.



Therefore, ∠A = (2 × 36)° = 72° and ∠B = (3 × 36°) = 108°.

Also, ∠B + ∠C = 180° [Since, ∠B and ∠C are adjacent angles]

= 108° + ∠C = 180° [Since, ∠B = 108°]

∠C = (180° - 108°) = 72°.

Also, ∠C + ∠D = 180° [Since, ∠C and ∠D are adjacent angles]

⇒ 72° + ∠D = 180°

⇒ ∠D = (180° - 72°) 108°.

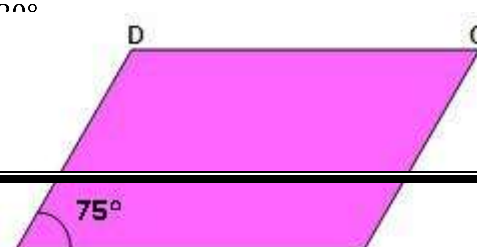
Therefore, ∠A = 72°, ∠B = 108°, ∠C = 72°and ∠D = 108°

3. In the adjoining figure, ABCD is a parallelogram in which ∠A = 75°. Find the measure of each of the angles ∠B, ∠C and ∠D.

Ans. It is given that ABCD is a parallelogram in which ∠A = 75°.

Since the sum of any two adjacent angles of a parallelogram is 180°

∠A + ∠B = 180°



$\Rightarrow 75° + \angle B = 180°$

$\Rightarrow \angle B = (180° - 75°) = 105°$

Also, $\angle B + \angle C = 180°$ [Since, $\angle B$ and $\angle C$ are adjacent angles]

$\Rightarrow 105° + \angle C = 180°$

$\Rightarrow \angle C = (180° - 105°) = 75°$.

Further, $\angle C + \angle D = 180°$ [Since, $\angle C$ and $\angle D$ are adjacent angles]

$\Rightarrow 75° + \angle D = 180°$

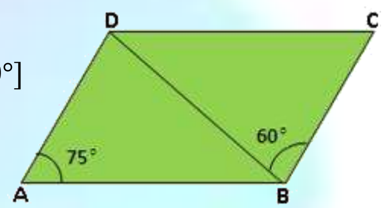
$\Rightarrow \angle D = (180° - 75°) = 105°$.

Therefore, $\angle B = 105°$, $\angle C = 75°$ and $\angle D = 105°$.

4. In the adjoining figure, ABCD is a parallelogram in which $\angle BAD = 75°$ and $\angle DBC = 60°$. Calculate: (i) $\angle CDB$ and (ii) $\angle ADB$.

Ans. We know that the opposite angles of a parallelogram are equal.

Therefore, $\angle BCD = \angle BAD = 75°$.



(i) Now, in Δ BCD, we have

$\angle CDB + \angle DBC + \angle BCD = 180°$ [Since, sum of the angles of a triangle is 180°]

$\Rightarrow \angle CDB + 60° + 75° = 180°$

$\Rightarrow \angle CDB + 135° = 180°$

$\Rightarrow \angle CDB = (180° - 135°) = 45°$.

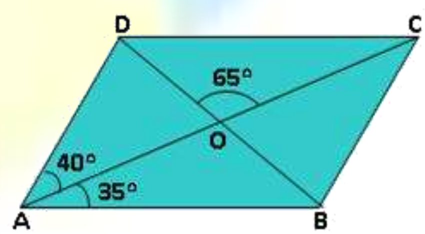
(ii) AD || BC and BD is the transversal.

Therefore, $\angle ADB = \angle DBC = 60°$ [alternate interior angles]

Hence, $\angle ADB = 60°$.

5. In the adjoining figure, ABCD is a parallelogram in which $\angle CAD = 40°$, $\angle BAC = 35°$ and $\angle COD = 65°$.

Calculate: (i) $\angle ABD$ (ii) $\angle BDC$ (iii) $\angle ACB$ (iv) $\angle CBD$.



Ans. (i) $\angle AOB = \angle COD = 65°$ (vertically opposite angles)

Now, in ΔOAB, we have:

$\angle OAB + \angle ABO + \angle AOB = 180°$ [Since, sum of the angles of a triangle is 180°]

$\Rightarrow 35° + \angle ABO + 65° = 180°$

$\Rightarrow \angle ABO + 100° = 180°$

$\Rightarrow \angle ABO = (180° - 100°) = 80°$

$\Rightarrow \angle ABD = \angle ABO = 80°$.

(ii) AB || DC and BD is a transversal.

Therefore, $\angle BDC = \angle ABD = 80°$ [alternate interior angles]

Hence, ∠BDC = 80°.

(iii) AD || BC and AC is a transversal.

Therefore, ∠ACB = ∠CAD = 40° [alternate interior angles]

Hence, ∠ACB = 40°.

(iv) ∠BCD = ∠BAD = (35° + 40°) = 75° [opposite angles of a parallelogram]

Now, in ΔCBD, we have

∠BDC + ∠BCD + ∠CBD = 180° [sum of the angles of a triangle is 180°]

⇒ 80° + 75° + ∠CBD = 180°

⇒ 155° + ∠CBD = 180°

⇒ ∠CBD = (180° - 155°) = 25°.

Hence, ∠CBD = 25°.

6. In the adjoining figure, ABCD is a parallelogram, AO and BO are the bisectors of ∠A and ∠B respectively. Prove that ∠AOB = 90°.

Ans. We know that the sum of two adjacent angles of a parallelogram is 180°

Therefore, ∠A + ∠B = 180° ……………. (i)

Since AO and BO are the bisectors of ∠A and ∠B, respectively, we have

∠OAB = $\frac{1}{2}$∠A and ∠ABO = 1/2∠B.

From ΔOAB, we have

∠OAB + ∠AOB + ∠ABO = 180° [Since, sum of the angles of a triangle is 180°]

⇒ $\frac{1}{2}$∠A + ∠ABO + $\frac{1}{2}$∠B = 180°

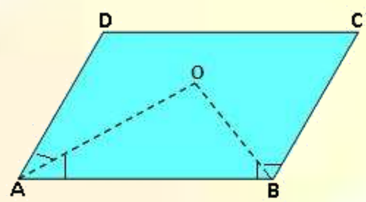
⇒ $\frac{1}{2}$ (∠A + ∠B) + ∠AOB = 180°

⇒ ($\frac{1}{2}$× 180°) + ∠AOB = 180° [using (i)]

⇒ 90° + ∠AOB = 180°

⇒ ∠AOB = (180° - 90°) = 90°.

Hence, ∠AOB = 90°.



7. The ratio of two sides of a parallelogram is 4 : 3. If its perimeter is 56 cm, find the lengths of its sides.

Ans. Let the lengths of two sides of the parallelogram be 4x cm and 3x cm respectively.

Then, its perimeter = 2(4x + 3x) cm = 8x + 6x = 14x cm.

Therefore, 14x = 56 ⇔ x = $\frac{56}{14}$ = 4.

Therefore, one side = (4 × 4) cm = 16 cm and other side = (3 × 4) cm = 12 cm.