Class –IX	Topic – INDICES

1. Evaluate :

$(i)\ 9^{\frac{3}{2}}$

$(ii)\ 8^{\frac{2}{3}}$

$(iii)\ 8^{-\frac{4}{3}}$

$(iv)\ (243)^{\frac{3}{5}}$

Solution:

$(i)\ 9^{\frac{3}{2}} = (3\times 3)^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^{1\times\frac{3}{1}}$ $\quad [\because (a^m)^n = a^{mn}]$

$= 3^3 = 3\times 3\times 3 = 27$

$(ii)\ 8^{\frac{2}{3}} = (2\times 2\times 2)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3\times\frac{2}{3}}$ $\quad [\because (a^m)^n = a^{mn}]$

$= 2^2 = 4$

$(iii)\ 8^{-\frac{4}{3}} = (2\times 2\times 2)^{-\frac{4}{3}} = (2^3)^{-\frac{4}{3}} = 2^{3\times\frac{-4}{3}}$ $\quad [\because (a^m)^n = a^{mn}]$

$= 2^{-4} = \frac{1}{2^4} = \frac{1}{2\times 2\times 2\times 2} = \frac{1}{16}$ $\quad \left[\because a^{-n} = \frac{1}{a^n}\right]$

$(iv)\ (243)^{-\frac{3}{5}} = (3\times 3\times 3\times 3\times 3)^{-\frac{3}{5}} = (3^5)^{-\frac{3}{5}} = 3^{5\times\frac{-3}{5}}$

$= (3)^{-3} = \frac{1}{3^3} = \frac{1}{3\times 3\times 3} = \frac{1}{27}$ $\quad \left[\because a^{-n} = \frac{1}{a^n}\right]$

2. Simplify :

$(i)\ \dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

$(ii)\ \dfrac{3^{n+1}}{3^{n(n-1)}} \div \dfrac{9^{n+1}}{(3^{n+1})^{n-1}}$

$(iii)\ (a+b)^{-1}.(a^{-1}+b^{-1})$

$(iv)\ \dfrac{5^{n+3} - 6\times 5^{n+1}}{9\times 5^n - 5^n\times 2^2}$

Solution:

$(i)\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}=\dfrac{3^n\times (3^2)^{n+1}}{3^{n-1}\times (3^2)^{n-1}}=\dfrac{3^n\times 3^{2n+2}}{3^{n-1}\times 3^{2n-2}}$

$=\dfrac{3^{n+2n+2}}{3^{n-1+2n-2}}=\dfrac{3^{3n+2}}{3^{3n-3}}$
$=3^{3n+2-3n+3}=3^5$

$(ii)\dfrac{3^{n+1}}{3^{n(n-1)}}\div\dfrac{9^{n+1}}{(3^{n+1})^{n-1}}=\dfrac{3^{n+1}}{3^{n(n-1)}}\times\dfrac{(3^{n+1})^{n-1}}{9^{n+1}}=\dfrac{3^{n+1}}{3^{n^2-n}}\times\dfrac{3^{(n+1)(n-1)}}{(3\times 3)^{n+1}}$

$=\dfrac{3^{n+1}}{3^{n^2-n}}\times\dfrac{3^{(n)^2-(1)^2}}{(3^2)^{n+1}}=\dfrac{3^{n+1}}{3^{n^2-n}}\times\dfrac{3^{n^2-1}}{3^{2(n+1)}}=\dfrac{3^{n+1}}{3^{n^2-n}}\times\dfrac{3^{n^2-1}}{3^{2n+2}}=3^{n+1+n^2-1-(n^2-n)-(2n+2)}$

$=3^{n+1+n^2-1-n^2+n-2n-2}=3^{-2}=\dfrac{1}{3^2}=\dfrac{1}{3\times 3}=\dfrac{1}{9}$

$(iii)\ (a+b)^{-1}.(a^{-1}+b^{-1})=\dfrac{1}{(a+b)}.\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$

$=\dfrac{1}{(a+b)}.\left(\dfrac{1\times b+1\times a}{ab}\right)=\dfrac{1}{(a+b)}.\dfrac{(b+a)}{ab}=\dfrac{1}{(a+b)}.\dfrac{(a+b)}{ab}=\dfrac{1}{ab}$

$(iv)\dfrac{5^{n+3}-6\times 5^{n+1}}{9\times 5^n-5^n\times 2^2}=\dfrac{5^{n+1}.5^2-6\times 5^{n+1}}{9\times 5^n-5^n\times 4}=\dfrac{5^{n+1}(5^2-6)}{5^n(9-4)}$

$=\dfrac{5^n.5^1(25-6)}{5^n(5)}=\dfrac{5^n.5(19)}{5^n(5)}=19$

3. If $1960=2^a.5^b.7^c$, calculate the value of $2^{-a}.7^b.5^{-c}$

Solution:

$1960=2^a.5^b.7^c\Rightarrow 2^a.5^b.7^c=1960\Rightarrow 2^a.5^b.7^c=2\times 2\times 2\times 2\times 5\times 7\times 7\Rightarrow 2^a.5^b.7^c$
$=2^3\times 5^1\times 7^2\quad ...(1) \mathit{Comparing\ powers\ of}\ 2,5\ \mathit{and}\ 7\ \mathit{on\ both\ sides\ of\ equation}\ (1), \mathit{we\ get} a$
$=3, b=1, c=2 \mathit{Hence, the\ value\ of}: 2^{-a}.7^b.5^{-c}=2^{-3}.7^1.5^{-2}=\dfrac{1}{2^3}\times 7\times\dfrac{1}{5^2}=\dfrac{7}{2^3\times 5^2}$
$=\dfrac{7}{2\times 2\times 2\times 5\times 5}=\dfrac{7}{8\times 25}=\dfrac{7}{200}$

4. Evaluate :

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1}=\frac{27}{8}$$

Solution:

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8} = \frac{(3)^3}{(2)^3} = \left(\frac{3}{2}\right)^3 \Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3} \textit{Comparing both sides, we get} \frac{x-1}{3} = -3$$

$$\Rightarrow x - 1 = -9$$

$$\Rightarrow x = -9 + 1 = -8 Hence, x = -8$$

5. Solve for x:
   (i) $4^{x-2} - 2^{x+1} = 0$
   (ii) $3^{x^2} : 3^x = 9 : 1$
   (iii) $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$

Solution:

$(i)\ 4^{x-2} - 2^{x+1} = 0$

$\Rightarrow 4^{x-2} = 2^{x+1} \Rightarrow 4^{x-2} = 2^{x+1}$

$\Rightarrow (2 \times 2)^{x-2} = 2^{x+1}$

$\Rightarrow 2^{2(x-2)} = 2^{x+1}$

$\Rightarrow 2^{2x-4} = 2^{x+1} \Rightarrow 2x - 4 = x + 1$

$\Rightarrow 2x - x = 1 + 4$

$\Rightarrow x = 5$

$(ii)\ 3^{x^2} : 3^x = 9 : 1$

$$\Rightarrow \frac{3^{x^2}}{3^x} = \frac{9}{1} \Rightarrow 3^{x^2 - x} = 9 \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right] \Rightarrow 3^{x^2-x} = 3 \times 3 \Rightarrow 3^{x^2-x} = 3^2 \Rightarrow x^2 - x$$
$$= 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x^2 - 2x + x - 2 = 0 \Rightarrow x(x-2) + 1(x-2) = 0$$
$$\Rightarrow (x+1)(x-2) = 0.$$

$If\ (x-2) = 0\ then\ x = 2. If\ (x+1) = 0\ then\ x = -1$

$(iii)\ 8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x \Rightarrow 8 \times (2^x)^2 + 4 \times 2^x \times 2^1 = 1 + 2^x$

$\Rightarrow 8\times(2^x)^2+8\times 2^x=1+2^x \Rightarrow 8\times(2^x)^2+8\times 2^x-2^x-1=0 \Rightarrow 8\times(2^x)^2+2^x(8-1)-1 = 0$

$\Rightarrow 8\left(2^{x^2}\right)+7\times 2^x-1=0[Putting\ 2^x=y] \Rightarrow 8\times(y^2)+7\times y-1=0 \Rightarrow 8y^2+7y-1=0$
$\Rightarrow 8y^2+8y-y-1=0 \Rightarrow 8y(y+1)-1(y+1)=0 \Rightarrow (y+1)(8y-1)=0$
$\Rightarrow (y+1)=0\ and\ 8y-1=0 \Rightarrow 2^x+1=0\ and\ 8\times 2^x-1=0 \Rightarrow 2^x=-1$

$and\ 8\times 2^x=1 \Rightarrow 2^x=-1\ and\ 2^x=\frac{1}{8} \Rightarrow 2^x=-1\ and\ 2^x=\frac{1}{2^3} \Rightarrow 2^x=-1\ and\ 2^x=2^{-3}$

$\Rightarrow x=-3$

6. Prove that :

$$(i)\ \left(\frac{x^a}{x^b}\right)^{a+b}.\left(\frac{x^b}{x^c}\right)^{b+c}.\left(\frac{x^c}{x^a}\right)^{c+a}=1$$

$$(ii)\ \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}.\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}.\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}=1(iii)\ \left(\frac{x^a}{x^b}\right)^{(a+b-c)}.\left(\frac{x^b}{x^c}\right)^{(b+c-a)}.\left(\frac{x^c}{x^a}\right)^{(c+a-b)}=1$$

Solution:

$(i)\ LHS=\left(\frac{x^a}{x^b}\right)^{a+b}.\left(\frac{x^b}{x^c}\right)^{b+c}.\left(\frac{x^c}{x^a}\right)^{c+a}$

$=x^{(a-b)(a+b)}.x^{(b-c)(b+c)}.x^{(c-a)(c+a)}=x^{a^2-b^2}.x^{b^2-c^2}.x^{c^2-a^2}=x^{a^2-b^2+b^2-c^2+c^2-a^2}=x^0=1 = RHS$

$(ii)\ LHS=\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}.\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}.\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$=x^{(a-b)\frac{1}{ab}}.x^{(b-c)\frac{1}{bc}}.x^{(c-a)\frac{1}{ca}}=x^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ca}}=x^{\frac{c(a-b)+a(b-c)+b(c-a)}{abc}}=x^{\frac{ca-bc+ab-ca+bc-ab}{abc}}=x^0 = 1 = RHS$

$(iii)\ LHS=\left(\frac{x^a}{x^b}\right)^{(a+b-c)}.\left(\frac{x^b}{x^c}\right)^{(b+c-a)}.\left(\frac{x^c}{x^a}\right)^{(c+a-b)}$

$=x^{(a-b)(a+b-c)}.x^{(b-c)(b+c-a)}.x^{(c-a)(c+a-b)}x^{a^2-b^2-ac+bc+b^2-c^2-ab+ca+c^2-a^2-bc+ab}=x^0=1 = RHS$

7. If $a^x=b^y=c^z\ and\ b^2=ac$, prove that: $y=\frac{2xz}{x+z}$

Solution:

$$Let\ a^x = b^y = c^z = k \therefore a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}\ and\ c = k^{\frac{1}{z}} Now, b^2 = ac \Rightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}} \Rightarrow k^{\frac{2}{y}}$$
$$= k^{\frac{1}{x}+\frac{1}{z}}\ Comparing\ both\ sides, we\ get \frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow \frac{2}{y} = \frac{x+z}{xz} \Rightarrow y(x+z)$$
$$= 2xz\ Hence, y = \frac{2xy}{x+z}$$

8. **If** $2^x = 3^y = 12^z$ **, show that :** $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$

Solution:

$$Let\ 2^x = 3^y = 12^z = k, then 2^x = k \Rightarrow 2 = k^{\frac{1}{x}}, 3^y = k \Rightarrow 3 = k^{\frac{1}{y}} 12^z = k \Rightarrow 2 = k^{\frac{1}{z}}. Now, 12$$
$$= 2 \times 2 \times 3 = 2^2 \times 3 \Rightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^2 \times k^{\frac{1}{y}} \Rightarrow k^{\frac{1}{z}} = k^{\frac{2}{x}} \times k^{\frac{1}{y}} \Rightarrow k^{\frac{1}{z}}$$
$$= k^{\frac{2}{x}+\frac{1}{y}} Comparing\ both\ sides, we\ get \frac{1}{z} = \frac{2}{x} + \frac{1}{y}$$

9. **If** $2^x = 3^y = 6^{-z}$ **, show that :** $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Solution:

$$Let\ 2^x = 3^y = 6^{-z} = k, then\ 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{-\frac{1}{z}} Now\ 2 \times 3 = 6 \Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}} \Rightarrow k^{\frac{1}{x}+\frac{1}{y}}$$
$$= k^{-\frac{1}{z}} Comparing\ both\ sides, we\ get \frac{1}{x} + \frac{1}{y} = -\frac{1}{z} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

10. **If** $a^x = b, b^y = c\ and\ c^z = a$, **prove that: xyz = 1.**

Solution:

$$We\ are\ given\ that\ a^x = b, b^y = c\ and\ c^z = a a^x = b \Rightarrow a^{xyz}$$
$$= b^{yz} \qquad (Raising\ to\ the\ power\ yz\ both\ sides) \Rightarrow a^{xyz} = (b^y)^z$$
$$= c^z \qquad (\because b^y = c) \Rightarrow a^{xyz} = a \qquad (\because c^z = a) \Rightarrow a^{xyz}$$
$$= a^1 Comparing\ both\ sides, we\ get \Rightarrow xyz = 1\ or\ a = c^z = (b^y)^z$$
$$= b^{yz} \qquad (\because c = b^y) = (a^x)^{yz} = a^{xyz} \qquad (\because b$$
$$= a^x Comparing\ both\ sides, we\ get xyz = 1$$