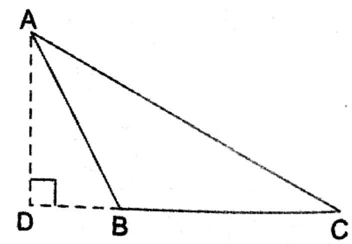
Class – 9th	Topic – Pythagoras theorem

1. **In ΔABC, obtuse angled at B, if AD is perpendicular to CB produced, prove that: $AC^2 = AB^2 + BC^2 + 2BC \times BD$**



Solution:

In $\Delta ADB, \angle D = 90°$

$\therefore AD^2 + DB^2 = AB^2$ ... (i) [By Pythagoras theorem]

In $\Delta ADC, \angle D = 90°$

$\therefore AC^2 = AD^2 + DC^2$ [By Pythagoras theorem]

$= AD^2 + (DB + BC)^2$

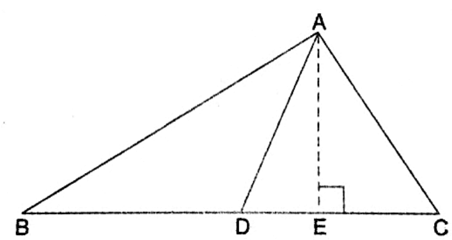
$= AD^2 + DB^2 + BC^2 + 2DB \times BC$

$= AB^2 + BC^2 + 2BC \times BD$ [Using (i)]

Hence, $AC^2 = AB^2 + BC^2 + 2BC \times BD$

2. **In ΔABC, if AD is the median, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$**

Solution:



Draw$AE \perp BC$. Then, $\angle AED = 90°$

$\therefore \angle ADE < 90°$

$\Rightarrow \angle ADB$ is an obtuse angle

thus, $\angle ADB$ is obtuse and $\angle ADE$ is acute.

now, ΔABD is obtuse-angled at D and $AE \perp BD$ produced.

$\therefore AC^2 = AD^2 + DC^2 - 2DC \times DE$ ... (i)

Also, ΔADC is acute-angled at D and $AE \perp DC$

$\therefore AC^2 = AD^2 + DC^2 - 2DC \times DE$

$= AD^2 + BD^2 - 2BD \times DE$ ... (ii) [$\because DC = BD$]

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2AD^2 + 2BD^2$

$= 2AD^2 + 2\left(\frac{1}{2}BC^2\right)$

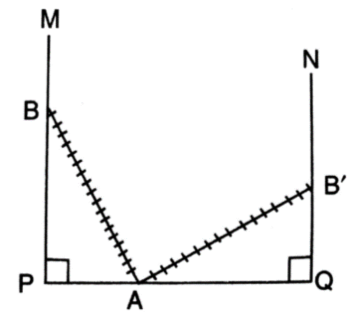
$= 2AD^2 + \frac{1}{2}BC^2$

Hence, $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$

3. **A ladder reached a window which is 15 metres above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 8 metre high. Find the width of the street, if the length of the ladder is 17 metres.**

Solution:

According to the given statement, the figure will be as shown alongside. In the figure, PQ is the width of the street, PM and QN are two building, A is the foot of the ladder, AB Is the first position of the ladder and AB' is its second position.



Clearly, $AB = AB' = 17\text{ m}, PB = 15\text{m and } QB' = 8\text{m}$

In $\Delta PAB, PA^2 + PB^2 = AB^2$

$\Rightarrow PA^2 + 15^2 = 17^2$

i. e. $PA^2 = 17^2 - 15^2 = 289 - 225 = 64 \ \Rightarrow PA = \sqrt{64}\text{ m} = 8\text{m}$

In $\Delta QAB', QA^2 + (QB')^2 = (AB')^2$

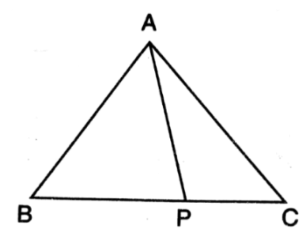
$\Rightarrow QA^2 + 8^2 = 17^2$

i. e. $QA^2 = 17^2 - 8^2 = 289 - 64 = 225 \ \Rightarrow QA = \sqrt{225}\text{ m} = 15\text{m}$

$\therefore$ The width of the street $= PQ$

$= PA + QA = 8\text{m} + 15\text{m} = 23\text{m}$

4. **ABC is an equilateral triangle; P is a point in BC such that BP: PC = 2:1.**
Prove that : $9AP^2 = 7AB^2$

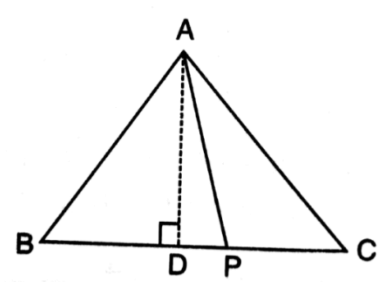


Solution:

According to the given statement, the figure will be as shown alongside.

Draw AD perpendicular to BC. In order to find AP, which is a side of right ΔADP; find DP and AD.

To find DP:



Given: $BP : PC = 2:1$

$\Rightarrow BP = \frac{2}{3} BC = \frac{2}{3} AB \qquad [\because AB = BC = AC]$

$AD \perp BC \ \Rightarrow BD = \frac{1}{2} BC = \frac{1}{2} AB$

$\therefore DP = BP - BD$

$= \frac{2}{3} AB - \frac{1}{2} AB = \frac{4AB - 3AB}{6} = \frac{AB}{6}$

To find AD: In right ΔABD,

$$AD^2 + BD^2 = AB^2 \Rightarrow AD^2 + \left(\frac{AB}{2}\right)^2 = AB^2 \qquad \left[\because BD = \frac{1}{2}BC = \frac{1}{2}AB\right]$$

$$\Rightarrow AD^2 = AB^2 - \frac{AB^2}{4} = \frac{3AB^2}{4}$$

Now, in right ΔADP,

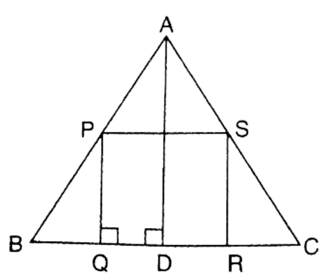
$$AP^2 = AD^2 + DP^2 \quad \Rightarrow AP^2 = \frac{3AB^2}{4} + \left(\frac{AB}{6}\right)^2$$

$$\text{i.e. } AP^2 = \frac{3AB^2}{4} + \frac{AB^2}{36} = \frac{27AB^2 + AB^2}{36}$$

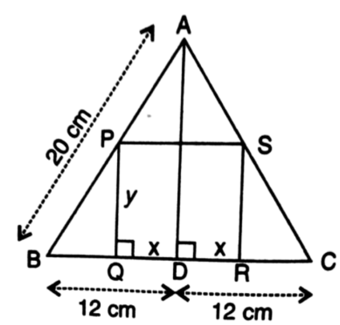
$$\text{i.e. } AP^2 = \frac{28AB^2}{36} = \frac{7AB^2}{9} \quad \Rightarrow 9AP^2 = 7AB^2$$

5. **ABC is an isosceles triangle in which $AB = AC = 20$ cm and $BC = 24$ cm. PQRS is a rectangle drawn inside the isosceles triangle. Given $PQ = SR = y$ cm and $PS = QR = 2x$ cm.**

Prove that: $y = 16 - \dfrac{4x}{3}$



Solution:



In an isosceles triangle, the perpendicular form vertex bisects the base.

$$\therefore BD = CD = \frac{24}{2}\text{ cm} = 12\text{ cm}$$

By ASA, ΔPBQ ≡ ΔSCR

$\Rightarrow BQ = CR$

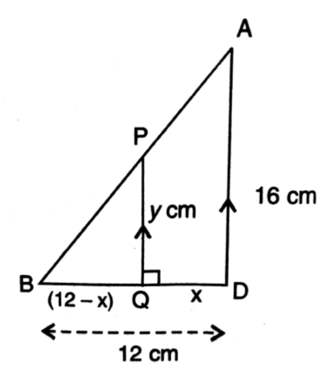
$$\therefore BD - BQ = CD - CR \Rightarrow DQ = DR = \frac{2x}{2}\text{ cm} = x\text{ cm and,}$$

$$BQ = CR = (12 - x)\text{ cm}$$

In right ΔABD, $AD^2 = AB^2 - BD^2$ [Using Pythagoras theorem]

$= 20^2 - 12^2 = 256$

$\Rightarrow AD = 16$ cm



Since, PQ and AD both are perpendicular to the same line BD;

therefore, PQ is parallel to AD.

In ΔABD, PQ||AD

$\Rightarrow \frac{PQ}{AD} = \frac{BQ}{BD} \Rightarrow \frac{y}{16} = \frac{12-x}{12}$

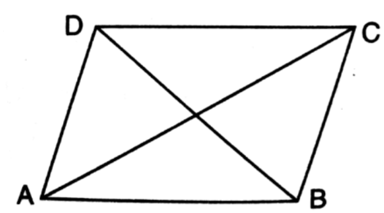
$\Rightarrow y = 16 - \frac{4x}{3}$

Hence proved

6. **Prove that the sum of the squares on the diagonals of a parallelogram is equal to the sum of the squares on its sides.**

Solution:

According to the given statement, the figure will be as shown alongside.



Required to prove:

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Draw AN ⊥ CD (Produced) and DM ⊥ AB

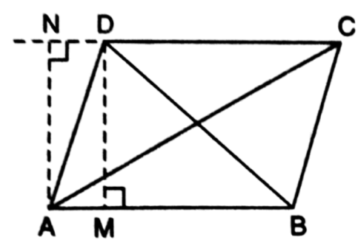
In right triangle ANC,

$AC^2 = AN^2 + NC^2$

$= AN^2 + (CD + DN)^2$

$= AN^2 + CD^2 + DN^2 + 2CD \times DN$

$= (AN^2 + DN^2) + CD^2 + 2CD \times DN$



$= AD^2 + CD^2 + 2CD \times DN$ ... (i)

In right ΔDMB,

$BD^2 = DM^2 + MB^2$

$= DM^2 + (AB - AM)^2$

$= DM^2 + AB^2 + AM^2 - 2AB \times AM$

$= (DM^2 + AM^2) + AB^2 - 2CD \times DN$

$= AD^2 + AB^2 - 2CD \times DN$ ...(ii)

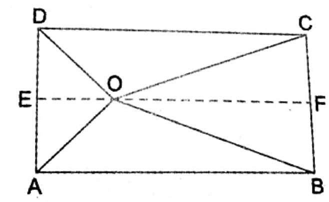
On adding equations (i) and (ii) we get

$AC^2 + BD^2 = AD^2 + CD^2 + AD^2 + AB^2$

$= AB^2 + BC^2 + CD^2 + DA^2$

Hence proved

7. **A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. prove that: $\mathbf{OA^2 + OC^2 = OB^2 + OD^2}$.**



Solution:

Through O, draw EOF || AB. Then, ABFE is a rhombus.

In right triangles OEA and OFC, we have:

$OA^2 = OE^2 + AE^2$

$OC^2 = OF^2 + CF^2$

$\therefore OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ .... (i)

Again, in right triangles OFB and OED, we have: $OB^2 = OF^2 + BF^2$

$OD^2 = OE^2 + DE^2$

$\therefore OB^2 + OD^2 = OF^2 + OE^2 + BF^2 + DE^2$

$= OE^2 + OF^2 + AE^2 + CF^2$ ... (ii)

From (i) and (ii), we get

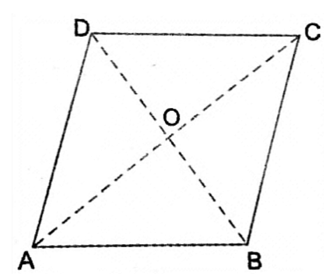
$OA^2 + OC^2 = OB^2 + OD^2$

8. **In a rhombus ABCD, prove that: $\mathbf{AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2}$**

Solution:

Let ABCD be a rhombus, whose diagonals AC and BD intersects at O. we know that the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$ and $AO = OC, BO = OD$



From right ΔAOB, we have $AB^2 = OA^2 + OB^2$

$= \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = \frac{1}{4}AC^2 + \frac{1}{4}BD^2$

$\Rightarrow 4AB^2 = AC^2 + BD^2$ .... (i)

Similarly, $4BC^2 = AC^2 + BD^2$

$4CD^2 = AC^2 + BD^2$

$4AD^2 = AC^2 + BD^2$

$\therefore 4(AB^2 + BC^2 + CD^2 + AD^2) = 4(AC^2 + BD^2)$

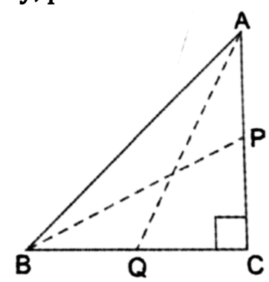
$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

9. In a ΔABC, right angled at C, if P and Q be the mid-points of CA and CB respectively, prove that:

(i) $4AQ^2 = 4AC^2 + BC^2$

(ii) $4BP^2 = 4BC^2 + AC^2$

(iii) $4(AQ^2 + BP^2) = 5AB^2$



Solution:

(i) From right ΔACQ, we have

$AQ^2 = AC^2 + QC^2$

$= AC^2 + \left(\frac{1}{2} BC\right)^2 = AC^2 + \frac{1}{4} AC^2$

$\Rightarrow 4AQ^2 = 4AC^2 + BC^2$

(ii) From right ΔPCB, we have $BP^2 = BC^2 + PC^2$

$= BC^2 + \left(\frac{1}{2} AC\right)^2 = BC^2 + \frac{1}{4} AC^2$

$\Rightarrow 4BP^2 = 4BC^2 + AC^2$

(iii) Adding (i) and (ii) we get

$4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$

$= 5AB^2 \quad [\because AC^2 + BC^2 = AB^2]$

Hence, $4(AQ^2 + BP^2) = 5AB^2$

10. In $\Delta ABC, \angle A = 90°$ and $AD \perp BC$. Prove that: $AD^2 = BD \times DC$.

Solution:

From right ΔADB, we have,

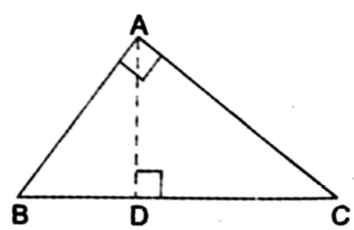
$AB^2 = AD^2 + BD^2 \quad \text{.... (i)}$

From right ΔADC, we have

$AC^2 = AD^2 + DC^2 \quad \text{... (ii)}$

From right ΔBAX, we have

$BC^2 = AB^2 + AC^2 \quad \text{.... (iii)}$



Adding (i)and (ii), we get

$AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$\Rightarrow BC^2 = 2AD^2 + BD^2 + DC^2$

$\Rightarrow (BD + DC)^2 = 2AD^2 + BD^2 + DC^2$

$\Rightarrow BD^2 + DC^2 + 2BD \times DC = 2AD^2 + BD^2 + DC^2$

$\Rightarrow AD^2 = BD \times DC$