SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

QUADRILATERALS

Exercise- 8.1

Q.1 The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all angles of the quadrilateral.

Ans - Let in quadrilateral ABCD, $\angle A = 3x$, $\angle B = 5x$, $C = 9x$ and $D = 13x$.

Since, sum of all the angles of a quadrilateral $= 360°$

$\therefore\ \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Now $\angle A = 3x = 3 \times 12 = 36°$

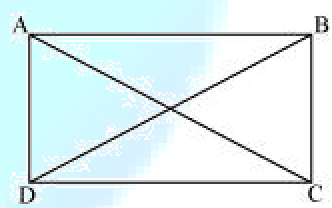
$\angle B = 5x = 5 \times 12 = 60°$

$\angle C = 9x = 9 \times 12 = 108°$

And $\angle D = 12x = 13 \times 12 = 156°$

Hence angles of given quadrilateral are $36°, 60°, 108°$ and $156°$

Q.2 If the diagonals of a parallelogram are equal, show that it is a rectangle.



Ans - Given: ABCD is a parallelogram with diagonal $AC =$ diagonal BD

To prove: ABCD is a rectangle.

Proof: In triangles ABC and ABD,

$AB = AB$[Common]

$AC = BD$[Given]

$AD = BC$[opp. Sides of a $\parallel$gm]

$\therefore\ \Delta\, ABC \cong \Delta\, BAD$ [By SSS congruency] $\Rightarrow \angle DAB = \angle CBA$ [By C. P. C. T.] ... ... (i)

But $\angle DAB + \angle CBA = 180°$ ... ... (ii)

[$\because$ AD$\parallel$ BC and AB cuts them, the sum of the interior angles of the same side of transversal is $180°$]

From eq. (i)and (ii),

$\angle DAB = \angle CBA = 90°$

Hence ABCD is a rectangle.

Q.3 Show that is diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

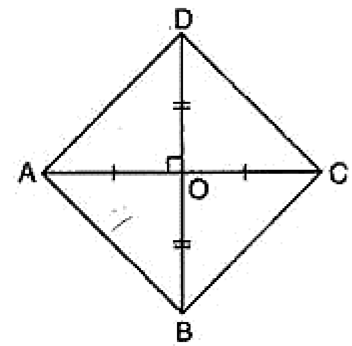
Ans - Given: Let ABCD is a quadrilateral.

Let its diagonal AC and BD bisect each other at right angle at point O.

$\therefore OA = OC, OB = OD$

and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$

To prove: ABCD is a rhombus.



Proof: In ΔAOD and ΔBOC,

$OA = OC$ [Given]$\therefore$ $\Delta AOD \cong \Delta COB$[By SAS congruency]

$\Rightarrow AD = CB$[By C. P. C. T.] ……… . (i)

Again, In ΔAOB and ΔCOD,

$OA = OC$[Given]

$\angle AOB = \angle COD$ [Given]

$OB = OD$[Given]

$\therefore \Delta AOB \cong \Delta COD$ [By SAS congruency]

$\Rightarrow AD = CB$ [By C. P. C. T.] ……… . (ii)

Now In ΔAOD and ΔBOC,

$OA = OC$[Given]

$\angle AOB = \angle BOC$ [Given]

$OB = OB$[Common]

$\therefore \Delta AOB \cong \Delta COB$ [By SAS congruency]

$\Rightarrow AB = BC$[By C.P.C.T.] ……….(iii)

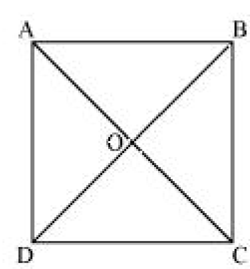
From eq. (i), (ii) and (iii),

$AD = BC = CD = AB$

And the diagonals of quadrilateral ABCD bisect each other at right angle.

Therefore, ABCD is a rhombus.

Q.4 Show that the diagonals of a square are equal and bisect each other at right angles.



Ans - Given: ABCD is a square. AC and BD are its diagonals bisect each other at point O.

To prove: $AC = BD$ and $AC \perp BD$ at point O.

Proof: In triangles ABC and BAD,

$AB = AB$[Common]

$\angle ABC = \angle BAD = 90°$

$BC = AD$ [Sides of a square]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS congruency]

⇒AC = BD[By C.P.C.T.]Hence proved

Now in triangles AOB and AOD,

AO = AO[Common]

AB = AD[Sides of a square]

OB = OD[Diagonals of a square bisect each other]

∴ ΔAOB ≅ Δ AOD [By SSS congruency]

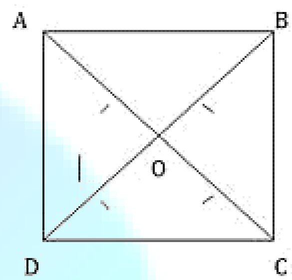
∠AOB = ∠AOD [By C.P.C.T.]

But ∠AOB + ∠AOD = 180° [Linear pair]

∴ ∠ AOB = ∠AOD = 90°

⇒ OA ⊥ BD or AC ⊥ BD Hence proved.Q.

Q.5 Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Ans - Let ABCD be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.

We have AC = BD and OA = OC……….(i)

And OB = OD……….(ii)

Now OA + OC = OB + OD

⇒ OC + OC = OB + OB [Using (i) & (ii)]

⇒ 2OC = 2OB

⇒ OC = OB……….(iii)

From eq. (i), (ii) and (iii), we get, OA = OB = OC = OD ……….(iv)

Now in ΔAOB and ΔCOD,

OA = OD [proved]

∠AOB = ∠COD[vertically opposite angles]

OB = OC [proved]

∴ ΔAOB ≅ ΔDOC[By SAS congruency]

⇒ AB = DC [By C.P.C.T.]……….(v)

Similarly, ΔBOC ≅ ΔAOD [By SAS congruency]

⇒ BC = AD [By C.P.C.T.]……….(vi)

From eq. (v) and (vi), it is concluded that ABCD is a parallelogram because opposite sides of a quadrilateral are equal.

Now in ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [proved above]

AC = BD [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SSS congruency]

$\Rightarrow$ $\angle ABC = \angle BAD$ [By C.P.C.T.] ....(vii)

But $\angle ABC + \angle BAD = 180^\circ$ [ABCD is a parallelogram] ....(viii)

$\therefore$ AD ∥ BC and AB is a transversal.

$\Rightarrow \angle ABC + \angle ABC = 180^\circ$ [Using eq. (vii) and (viii)]

$\Rightarrow 2\angle ABC = 180^\circ \Rightarrow \angle ABC = 90^\circ$

$\therefore \angle ABC = \angle BAD =$ ....90° (ix)

Opposite angles of a parallelogram are equal.

But $\angle ABC = \angle BAD =$

$\therefore \angle ABC = \angle ADC =$ ....90° (x)

$\therefore \angle BAD = \angle BDC =$ ....90° (xi)

From eq. (x) and (xi), we get

$\angle ABC = \angle ADC = \angle BAD = \angle BDC = 90^\circ$ ....(xii)

Now in ΔAOB and ΔBOC,

OA = OC [Given]

$\angle AOB = \angle BOC = 90^\circ$ [Given]

OB = OB [Common]

$\therefore$ $\Delta AOB \cong \Delta COB$ [By SAS congruency]

$\Rightarrow$ AB = BC..........(xiii)

From eq. (v), (vi) and (xiii), we get,

AB = BC = CD = AD ....(xiv)

Now, from eq. (xii) and (xiv), we have a quadrilateral whose equal diagonals bisect each other at right angle.

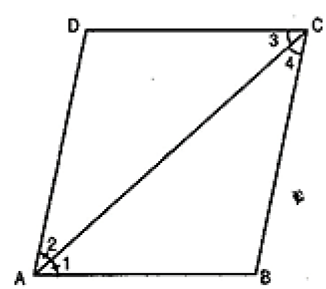
Also, sides are equal make an angle of 90° with each other.

$\therefore$ ABCD is a square.

Q.6 Diagonal AC of a parallelogram ABCD bisects $\angle A$ (See figure). Show that:

(i) It bisects $\angle C$ also.

(ii) ABCD is a rhombus,

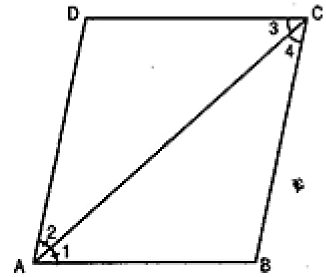


Ans - Diagonal AC bisects $\angle A$ of the parallelogram ABCD.

(i) Since AB ∥ DC and AC intersects them.

$\therefore \angle 1 = \angle 3$ [Alternate angles] ....(i)

Similarly, $\angle 2 = \angle 4$ ....(ii)



But $\angle 1 = \angle 2$ [Given]……….(iii)

$\therefore \angle 3 = \angle 4$[Using eq. (i), (ii) and (iii)]

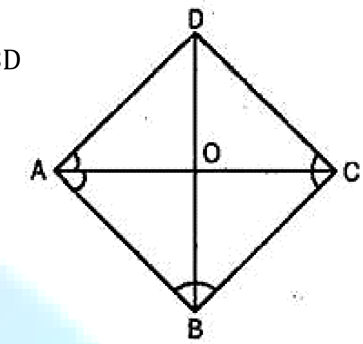
Thus, AC bisects $\angle C$.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$

$\Rightarrow$ AD = CD[Sides opposite to equal angles]

$\therefore$ AB = CD = AD = BC

Hence ABCD is a rhombus.



Q.7 ABCD is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans - ABCD is a rhombus. Therefore, AB = BC = CD = AD Let O be the point of bisection of diagonals.

$\therefore$ OA = OC and OB = OD

In ΔOB and ΔAOD,

OA = OA [Common]

AB = AD [Equal sides of rhombus]

OB = OD (diagonals of rhombus bisect each other]

$\therefore \Delta AOB \cong \Delta AOD$ [By SSS congruency]

$\Rightarrow \angle OAD = \angle OAB$ [By C.P.C.T.]

$\Rightarrow$ OA bisects $\angle A$ ……….(i)

Similarly, $\Delta BOC \cong \Delta DOC$ [By SSS congruency]

$\Rightarrow \angle OCB = \angle OCD$[By C.P.C.T.]

$\Rightarrow$ OC bisects $\angle C$ ……….(ii)

From eq. (i) and (ii), we can say that diagonal AC bisects $\angle A$ and $\angle C$.

Now in ΔAOB and ΔBOC,

OB = OB [Common]

AB = BC [Equal sides of rhombus]

OA = OC (diagonals of rhombus bisect each other]

$\therefore \Delta AOB \cong \Delta COB$ [By SSS congruency]

$\Rightarrow \angle OBA = \angle OBC$ [By C.P.C.T.]

$\Rightarrow$ OB bisects $\angle B$ ……….(iii)

Similarly, $\Delta AOD \cong \Delta COD$ [By SSS congruency]

$\Rightarrow \angle ODA = \angle ODC$[By C.P.C.T.]

$\Rightarrow$ BD bisects $\angle D$ ……….(iv)

From eq. (iii) and (iv), we can say that diagonal BD bisects $\angle B$ and $\angle D$.

Q.8 ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:

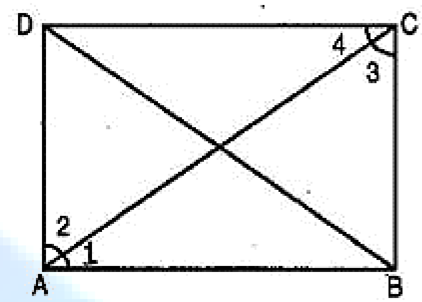
(i) ABCD is a square.

(ii) Diagonal BD bisects both ∠B as well as ∠D.

Ans - ABCD is a rectangle. Therefore AB = DC……….(i)

And BC = AD

Also ∠A = ∠B = ∠C = ∠D = 90°



(i) In ABC and ADC

∠1 = ∠2 and ∠3 = ∠4

[AC bisects ∠A and ∠C (given)]

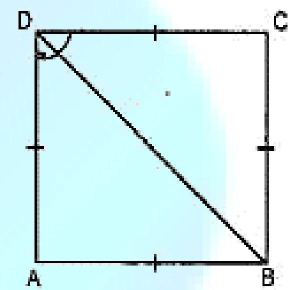
AC = AC [Common]

∴ Δ ABC ≅ ΔADC[By ASA congruency]

⇒ AB = AD ……….(ii)

From eq. (i) and (ii), AB = BC = CD = AD

Hence ABCD is a square.



(ii) In ΔABC andΔ ADC

AB = BA [Since ABCD is a square]

AD = DC [Since ABCD is a square]

BD = BD [Common]

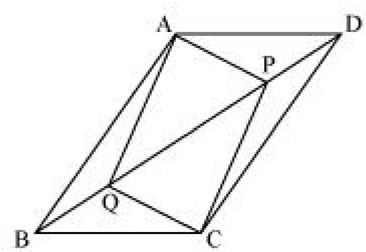
∴ ΔABD ≅ ΔCBD [By SSS congruency]

⇒ ∠ABD = ∠CBD [By C.P.C.T.]……….(iii)

And ∠ADB = ∠CDB[By C.P.C.T.]……….(iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both ∠B and ∠D.

Q.9 In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (See figure). Show that:



(i) ΔAPD ΔCQB

(ii) AP = CQ

(iii) ΔAQB≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram.

Ans - (i) In ΔAPD and ΔCQB,

DP = BQ [Given]

∠ADP = ∠QBC [Alternate angles (AD∥BC and BD are transversal)]

AD = CB [Opposite sides of parallelogram]

∴ $\Delta APD \cong \Delta CQB$ [By SAS congruency]

(ii) Since Δ APD$\cong$ ΔCQB

$\Rightarrow AP = CQ$ [By C.P.C.T.]

(iii) In ΔAQB and ΔCPD,

$BQ = DP$[Given]

$\angle ABQ = \angle PDC$ [Alternate angles ($AB \parallel CD$ and BD is transversal)]

$AB = CD$[Opposite sides of parallelogram]

∴ $\Delta AQB \cong \Delta CPD$ [By SAS congruency]

(iv) Since $\Delta AQB \cong \Delta CPD$

$\Rightarrow AQ = CP$[By C.P.C.T.]

(v) In quadrilateral APCQ,

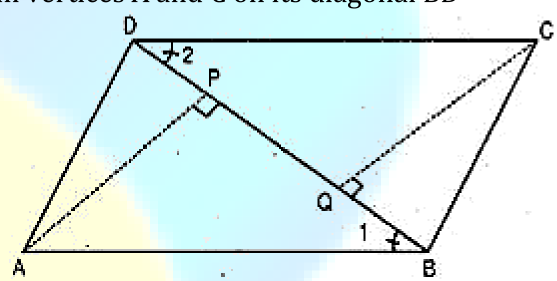
$AP = CQ$ [proved in part (i)]

$AQ = CP$ [proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

Q.10 ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:



(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Ans - Given ABCD is a parallelogram. $AP \perp BD$ and $CQ \perp BD$

To prove: (i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$

Proof:

(i) In ΔAPB and ΔCQD,

$\angle 1 = \angle 2$[Alternate interior angles]

$AB = CD$[Opposite sides of a parallelogram are equal]

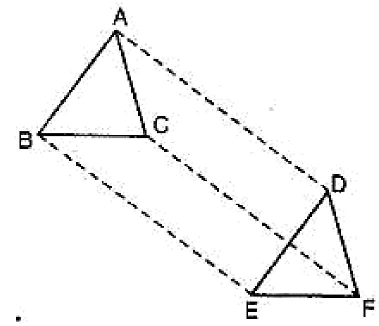
$\angle APB = \angle CQD = 90°$

∴ $\Delta APB \cong \Delta CQD$ [By ASA Congruency]

(ii) Since $\Delta APB \cong \Delta CQD$

∴ $AP = CQ$ [By C.P.C.T.]

Q.11 An ΔABC and ΔDEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:



(i) Quadrilateral ABED is a parallelogram.

(ii) Quadrilateral BEFC is a parallelogram.

(iii) AD ∥ CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

(v) AC = DF

(vi) ΔABC ≅ ΔDEF

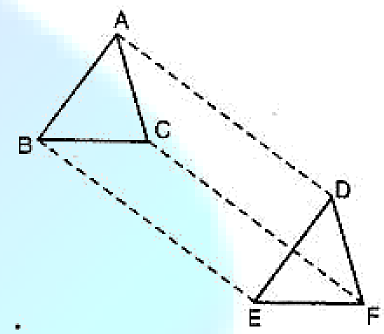
Ans - (i) In ΔABC and ΔDEF

AB = DE [Given]

And AB ∥ DE [Given]

∴ ABED is a parallelogram.

(ii)In ΔABC and ΔDEF



BC = EF [Given]

And BC ∥ EF [Given]

∴ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

∴ AD ∥ BE and AD = BE ……….(i)

Also, BEFC is a parallelogram.

∴ CF ∥ BE and CF = BE……….(ii)

From (i) and (ii), we get

∴ AD ∥ CF and AD = CF

(iv) As AD ∥ CF and AD = CF

⇒ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

∴ AC = DF

(vi) In ΔABC and ΔDEF,

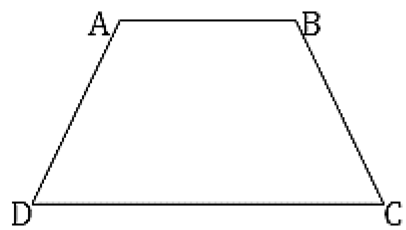
AB = DE [Given]

BC = EF [Given]

AC = DF [Proved]

∴ Δ ABC ≅ ΔDEF [By SSS congruency]

Q.12 ABCD is a trapezium in which AB ∥ CD and AD = BC (See figure). Show that:



(i) ∠A = ∠B

(ii) ∠C = ∠D

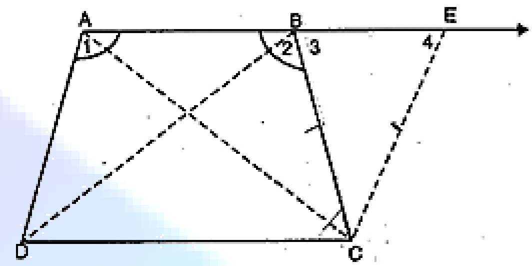
(iii) ΔABC ≅ ΔBAD

(iv) Diagonal AC = Diagonal BD

Ans - Given: ABCD is a trapezium.

AB || CD and AD = BC

To prove :



Construction: Draw CE ∥ AD and extend

AB to intersect CE at E.

Proof:

(i) As AECD is a parallelogram. [By construction]

∴ AD = EC

But AD = BC [Given]

∴ BC = EC

⇒ ∠3 = ∠4 [Angles opposite to equal sides are equal]

Now ∠1 + ∠4 = 180°[Interior angles]

And ∠2 + ∠3 = 180° [Linear pair]

⇒ ∠1 + ∠4 = ∠2 + ∠3

⇒ ∠1 = ∠2 [∵ ∠3 = ∠4]

⇒ ∠A = ∠B

(ii) ∠3 = ∠C [Alternate interior angles]

And ∠D = ∠4 [Opposite angles of a parallelogram]

But ∠3 = ∠4 [ΔBCE is an isosceles triangle]

∴ ∠C = ∠D

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

∠1 = ∠2 [Proved]

AD = BC [Given]

∴ ΔABC ≅ ΔBAD[By SAS congruency]

(iv) We had observed that,

∴ ΔABC ≅ ΔBAD

⇒AC = BD [By C.P.C.T.]