Board – ICSE | Class – 9th | Topic – Complementary Angles

1. Evaluate: $\frac{\cos\ 55^{\circ}}{\sin\ 35^{\circ}}+\frac{\cot\ 35^{\circ}}{\tan\ 55^{\circ}}$

SOLUTION

$$\begin{aligned}
&\frac{\cos\ 55^{\circ}}{\sin\ 35^{\circ}}+\frac{\cot\ 35^{\circ}}{\tan\ 55^{\circ}}\\
&=\frac{\cos\left(90^{\circ}-35^{\circ}\right)}{\sin\ 35^{\circ}}+\frac{\cot\left(90^{\circ}-55^{\circ}\right)}{\tan\ 55^{\circ}}\\
&=\frac{\sin\ 35^{\circ}}{\sin\ 35^{\circ}}+\frac{\tan\ 55^{\circ}}{\tan\ 55^{\circ}}\\
&=1+1\\
&=2
\end{aligned}$$

2. Evaluate: $\cos^{2}\ 25^{\circ}-\sin^{2}\ 65^{\circ}-\tan^{2}\ 45^{\circ}$

SOLUTION

$$\begin{aligned}
&\cos^{2}\ 25^{\circ}-\sin^{2}\ 65^{\circ}-\tan^{2}\ 45^{\circ}\\
&=\left[\cos\left(90^{\circ}-65^{\circ}\right)\right]^{2}-\sin^{2}\ 65^{\circ}-\left(\tan\ 45^{\circ}\right)^{2}\\
&=\sin^{2}\ 65^{\circ}-\sin^{2}\ 65^{\circ}-(1)^{2}\\
&=0-1\\
&=-1
\end{aligned}$$

3. Express the following in term of angles between 0° and 45° : $\sin\ 59^{\circ}+\tan\ 63^{\circ}$

SOLUTION

$$\begin{aligned}
&\sin\ 59^{\circ}+\tan\ 63^{\circ}\\
&=\sin\ (90-31)^{\circ}+\tan\ (90-27)^{\circ}\\
&=\cos\ 31^{\circ}+\cot\ 27^{\circ}
\end{aligned}$$

4. Express the following in term of angles between 0° and 45° : $\operatorname{cosec} 68^\circ + \cot 72^\circ$

SOLUTION

$\operatorname{cosec} 68^\circ + \cot 72^\circ$

$= \operatorname{cosec}(90-22)^\circ + \cot(90-18)^\circ$

$(\because \operatorname{cosec}(90-\theta) = \sec\theta$ and $\cot(90-\theta) = \tan\theta)$

$= \sec 22^\circ + \tan 18^\circ$

5. Express the following in term of angles between 0° and 45° : $\cos 74^\circ + \sec 67^\circ$

SOLUTION

$$\begin{aligned}&\cos 74^\circ + \sec 67^\circ\\&= \cos(90-16)^\circ + \sec(90-23)^\circ\\&= \sin 16^\circ + \operatorname{cosec} 23^\circ\end{aligned}$$

6. Evaluate: $3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

SOLUTION

$$\begin{aligned}&3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}\\&= 3\frac{\sin(90^\circ-18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ-58^\circ)}{\operatorname{cosec} 58^\circ}\\&= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ}\\&= 3-1\\&= 2\end{aligned}$$

7. Evaluate: $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\sin 59^\circ \sec 31^\circ$

SOLUTION

$$\begin{aligned}&3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\sin 59^\circ \sec 31^\circ\\&= 3\cos(90^\circ-10^\circ)\operatorname{cosec} 10^\circ + 2\sin(90^\circ-31)\sec 31^\circ\\&= 3\sin 10^\circ \operatorname{cosec} 10^\circ + 2\cos 31^\circ \sec 31^\circ\\&= 3\times 1 + 2\times 1\\&= 3+2\\&= 5\end{aligned}$$

8. Evaluate: $\frac{\sin\ 80^{\circ}}{\cos\ 10^{\circ}}+\sin\ 59^{\circ}\sec\ 31^{\circ}$

SOLUTION

$$\begin{aligned}
&\frac{\sin\ 80^{\circ}}{\cos\ 10^{\circ}}+\sin\ 59^{\circ}\sec\ 31^{\circ}\\
&=\frac{\sin\left(90^{\circ}-10^{\circ}\right)}{\cos\ 10^{\circ}}+\sin\left(90^{\circ}-31^{\circ}\right)\sec\ 31^{\circ}\\
&=\frac{\cos\ 10^{\circ}}{\cos\ 10^{\circ}}+\frac{\cos\ 31^{\circ}}{\cos\ 31^{\circ}}\\
&=1+1\\
&=2
\end{aligned}$$

9. Evaluate: $2\frac{\tan\ 57^{\circ}}{\cot\ 33^{\circ}}-\frac{\cot\ 70^{\circ}}{\tan\ 20^{\circ}}-\sqrt{2}\cos\ 45^{\circ}$

SOLUTION

$$\begin{aligned}
&2\frac{\tan\ 57^{\circ}}{\cot\ 33^{\circ}}-\frac{\cot\ 70^{\circ}}{\tan\ 20^{\circ}}-\sqrt{2}\cos\ 45^{\circ}\\
&=2\frac{\tan\left(90^{\circ}-33^{\circ}\right)}{\cot\ 33^{\circ}}-\frac{\cot\left(90^{\circ}-20^{\circ}\right)}{\tan\ 20^{\circ}}-\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\\
&=2\frac{\cot\ 33^{\circ}}{\cot\ 33^{\circ}}-\frac{\tan\ 20^{\circ}}{\tan\ 20^{\circ}}-1\\
&=2-1-1\\
&=0
\end{aligned}$$

10. Evaluate: $\frac{\cot^{2}\ 41^{\circ}}{\tan^{2}\ 49^{\circ}}-2\frac{\sin^{2}\ 75^{\circ}}{\cos^{2}\ 15^{\circ}}$

SOLUTION 1

$$\begin{aligned}
&\frac{\cot^{2}\ 41^{\circ}}{\tan^{2}\ 49^{\circ}}-2\frac{\sin^{2}\ 75^{\circ}}{\cos^{2}\ 15^{\circ}}\\
&=\frac{\left[\cot\left(90^{\circ}-49^{\circ}\right)\right]^{2}}{\tan^{2}\ 49^{\circ}}-2\frac{\left[\sin\left(90^{\circ}-15^{\circ}\right)\right]^{2}}{\cos^{2}\ 15^{\circ}}\\
&=\frac{\tan^{2}\ 49^{\circ}}{\tan^{2}\ 59^{\circ}}-2\frac{\cos^{2}\ 15^{\circ}}{\cos^{2}\ 15^{\circ}}\\
&=1-2\\
&=-1
\end{aligned}$$

11. Evaluate: $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

SOLUTION

$$\begin{aligned}&14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ\\&= 14\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right) - 5(1)\\&= 7 + 3 - 5\\&= 5\end{aligned}$$

12. A triangle ABC is right-angled at B; find the value of $\frac{\sec A\cdot\sin C-\tan A\cdot\tan C}{\sin B}$

SOLUTION

Since $\triangle$ ABC is a right angled triangle, right angled at B, $A + C = 90^\circ$

$$\therefore \frac{\sec A \cdot \sin C - \tan A \cdot \tan C}{\sin B}$$

$$= \frac{\sec A(90^\circ - C)\sin C - \tan(90^\circ - C)\tan C}{\sin 90^\circ}$$

$$= \frac{\operatorname{cosec} C.\sin C - \cot C.\tan C}{1}$$

$$= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C$$

$$= 1 - 1$$

$$= 0$$

13. In the case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

$\sin(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

SOLUTION

$\operatorname{Sin}(90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

$$\begin{aligned}&\Rightarrow \sin(90^\circ - 3A) = \frac{1}{\operatorname{cosec} 42^\circ}\\&\Rightarrow \cos 3A = \frac{1}{\operatorname{cosec}(90^\circ - 48^\circ)}\\&\Rightarrow \cos 3A = \frac{1}{\sec 48^\circ}\\&\Rightarrow \cos 3A = \cos 48^\circ\\&\Rightarrow 3A = 48^\circ\\&\Rightarrow A = 16^\circ\end{aligned}$$

14. In the case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

$\cos(90^\circ - A) \cdot \sec 77^\circ = 1$

SOLUTION

$$\cos(90^\circ - 3\,A) \cdot \sec 77^\circ = 1$$
$$\Rightarrow \cos(90^\circ - 3\,A) = \frac{1}{\sec 77^\circ}$$
$$\Rightarrow \sin 3\,A = \frac{1}{\sec(90 - 12^\circ)}$$
$$\Rightarrow \sin 3\,A = \frac{1}{\operatorname{cosec} 12^\circ}$$
$$\Rightarrow \sin 3\,A - \sin 12^\circ$$
$$\Rightarrow 3\,A = 12^\circ$$
$$\Rightarrow A = 3^\circ$$