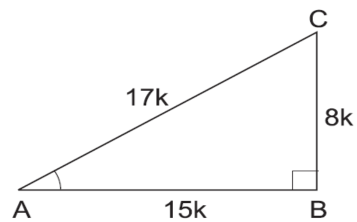
Board – ICSE | Class – 9 | Topic – Trigonometric Ratios Solved Examples

1. If $\sin A = \frac{8}{17}$, find other trigonometric ratios of $\angle A$.

Solution



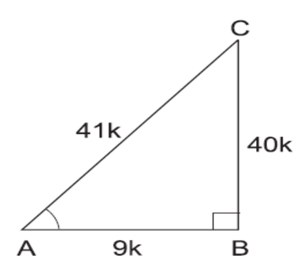
Let us draw a $\triangle ABC$ in which $\angle B = 90°$.
Then, $\sin A = \frac{BC}{AC} = \frac{8}{17}$.
Let $BC = 8k$ and $AC = 17k$, where k is positive.

By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow AB^2 = AC^2 - BC^2$$
$$\Rightarrow AB^2 = (17k)^2 - (8k)^2 = 289k^2 - 64k^2$$
$$= 225k^2$$
$$\Rightarrow AB = \sqrt{225k^2} = 15k$$
$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}; \cos A = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17};$$
$$\tan A = \frac{\sin A}{\cos A} = \left(\frac{8}{17} \times \frac{17}{15}\right) = \frac{8}{15}$$
$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{17}{8}; \sec A = \frac{1}{\cos A} = \frac{17}{15}$$
$$\text{and } \cot A = \frac{1}{\tan A} = \frac{15}{8}.$$

2. If $\cos A = \frac{9}{41}$, find other trigonometric ratios of $\angle A$.

Solution.



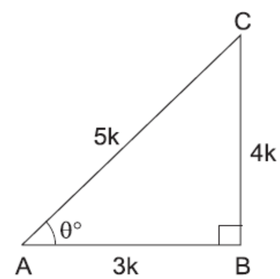
Let us draw a $\triangle ABC$ in which $\angle B = 90°$.
Then, $\cos A = \frac{AB}{AC} = \frac{9}{41}$.
Let $AB = 9k$ and $AC = 41k$, where k is positive.
By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow BC^2 = AC^2 - AB^2$$
$$\Rightarrow BC^2 = (41k)^2 - (9k)^2 = 1681k^2 - 81k^2 = 1600k^2$$
$$\Rightarrow BC = \sqrt{1600k^2} = 40k$$

$$\therefore\ \sin A = \frac{BC}{AC} = \frac{40k}{41k} = \frac{40}{41}; \cos A = \frac{9}{41}\ (\text{ given })$$
$$\tan A = \frac{\sin A}{\cos A} = \left(\frac{40}{41} \times \frac{41}{9}\right) = \frac{40}{9}$$
$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{41}{40}; \sec A = \frac{1}{\cos A} = \frac{41}{9}$$

and $\cot A = \frac{1}{\tan A} = \frac{9}{40}$.

3. If $\cos\theta = \frac{3}{5}$, find the value of $\left(\frac{5\operatorname{cosec}\theta - 4\tan\theta}{\sec\theta + \cot\theta}\right)$.



Solution

Let us draw a $\triangle ABC$ in which $\angle B = 90°$. Let $\angle A = \theta°$.
Then, $\cos\theta = \frac{AB}{AC} = \frac{3}{5}$.
Let $AB = 3k$ and $AC = 5k$, where k is positive.

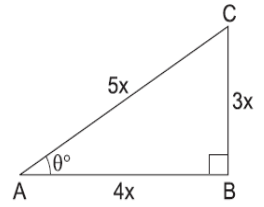
By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow BC^2 = AC^2 - AB^2 = (5k)^2 - (3k)^2 = 25k^2 - 9k^2 = 16k^2$$
$$\Rightarrow BC = \sqrt{16k^2} = 4k.$$
$$\therefore\ \sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}; \tan\theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3};$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}; \text{ and } \operatorname{cosec}\theta = \frac{AC}{BC} = \frac{5k}{4k} = \frac{5}{4}.$$
$$\therefore\ \left(\frac{5\operatorname{cosec}\theta - 4\tan\theta}{\sec\theta + \cot\theta}\right) = \frac{\left(5 \times \frac{5}{4} - 4 \times \frac{4}{3}\right)}{\left(\frac{5}{3} + \frac{3}{4}\right)} = \frac{\left(\frac{25}{4} - \frac{16}{3}\right)}{\left(\frac{5}{3} + \frac{3}{4}\right)}$$
$$= \frac{\frac{75-64}{12}}{\frac{20+9}{12}} = \left(\frac{11}{12} \times \frac{12}{29}\right) = \frac{11}{29}.$$

4. If $\sec\theta = \frac{5}{4}$, show that

$$\frac{(2\cos\theta - \sin\theta)}{(\cot\theta - \tan\theta)} = \frac{12}{7}$$

Solution



Consider a $\triangle ABC$ in which $\angle A = \theta$ and $\angle B = 90^\circ$.
$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB} = \frac{5}{4} = \frac{5x}{4x}$ (say)
$\therefore$ $AC = 5x$ and $AB = 4x$, where x is positive.
By Pythagoras' theorem we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad BC^2 = AC^2 - AB^2 = (5x)^2 - (4x)^2 = 9x^2$$
$$\Rightarrow \quad BC = 3x$$
$$\therefore \quad \cos\theta = \frac{AB}{AC} = \frac{4x}{5x} = \frac{4}{5}; \sin\theta = \frac{BC}{AC} = \frac{3x}{5x} = \frac{3}{5}$$

$$\cot\theta = \frac{AB}{BC} = \frac{4x}{3x} = \frac{4}{3};$$

and

$$\tan\theta = \frac{BC}{AB} = \frac{3x}{4x} = \frac{3}{4}$$

.

$$\therefore \frac{(2\cos\theta - \sin\theta)}{(\cot\theta - \tan\theta)} = \frac{\left(2\times\frac{4}{5} - \frac{3}{5}\right)}{\left(\frac{4}{3} - \frac{3}{4}\right)} = \frac{\left(\frac{8}{5} - \frac{3}{5}\right)}{\left(\frac{4}{3} - \frac{3}{4}\right)} = \frac{\left(\frac{5}{5}\right)}{\left(\frac{7}{12}\right)} = \frac{1}{\left(\frac{7}{12}\right)} = \frac{12}{7}$$

5. If $3\tan\theta = 4$, evaluate $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$

Solution

$$3\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$$

Given expression

$$= \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$$
$$= \frac{3\tan\theta + 2}{3\tan\theta - 2} \text{ [dividing num. and denom. by } \cos\theta]$$
$$= \frac{\left(3\times\frac{4}{3} + 2\right)}{\left(3\times\frac{4}{3} - 2\right)} = \frac{6}{2} = 3. \left[\because \tan\theta = \frac{4}{3}\right]$$

6. If $5\cot\theta = 3$, find the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$.

Solution

$5\cot\theta = 3 \Rightarrow \cot\theta = \frac{3}{5}$

Given expression $= \left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right) = \frac{(5 - 3\cot\theta)}{(4 + 3\cot\theta)}$
[dividing num. and denom. by $\sin\theta$]

$$= \frac{\left(5 - 3 \times \frac{3}{5}\right)}{\left(4 + 3 \times \frac{3}{5}\right)} = \frac{\left(5 - \frac{9}{5}\right)}{\left(4 + \frac{9}{5}\right)} = \left(\frac{16}{5} \times \frac{5}{29}\right) = \frac{16}{29}$$

7. If $7\sin^2\theta + 3\cos^2\theta = 4$, show that $\tan\theta = \frac{1}{\sqrt{3}}$.

Solution

$$7\sin^2\theta + 3\cos^2\theta = 4$$
$$\Rightarrow 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$$
$$\Rightarrow 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$$

$$\Rightarrow 4\sin^2\theta + 3 \times 1 = 4 \ [\because \sin^2\theta + \cos^2\theta = 1]$$
$$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin^2\theta = \frac{1}{4}$$
$$\therefore \cos^2\theta = (1 - \sin^2\theta) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}$$
$$\therefore \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \left(\frac{1}{4} \times \frac{4}{3}\right) = \frac{1}{3}$$

Hence, $\tan\theta = \frac{1}{\sqrt{3}}$

8. If $\cot\theta = \frac{15}{8}$ then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

Solution

$$\text{Given expression} = \frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$$
$$= \frac{2(1 + \sin\theta)(1 - \sin\theta)}{2(1 + \cos\theta)(1 - \cos\theta)}$$
$$= \frac{(1 - \sin^2\theta)}{(1 - \cos^2\theta)} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$
$$= (\cot\theta)^2 = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$$

Hence, the value of the given expression is $\frac{225}{64}$.

9. If cosec $\theta = \sqrt{5}$, find the value of: $2 - \sin^2\ \theta - \cos^2\ \theta$

Solution

$$\text{cosec}\ \theta = \sqrt{5}$$
$$\text{i.e.}\ \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\sqrt{5}}{1}$$

Therefore, if length of hypotenuse $= \sqrt{5}x$, length of perpendicular $= x$

Since

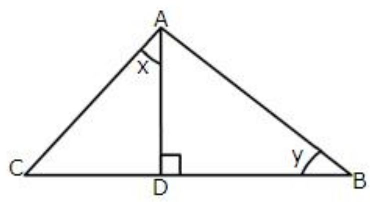
$$\begin{aligned}
&\text{base}^{\ 2} + \text{perpendicular}^{\ 2} = \text{hypotenuse}^2\\
&\text{base}^{\ 2} + (x)^2 = (\sqrt{5}x)^2\\
&\text{base}^{\ 2} = 5x^2 - x^2 = 4x^2\\
&\therefore\ \text{base} = 2x
\end{aligned}$$

Now

$$\sin\ \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}}$$
$$\cos\ \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{2x}{\sqrt{5}x} = \frac{2}{\sqrt{5}}$$

$$\begin{aligned}
\Rightarrow\quad & 2 - \sin^2\ \theta - \cos^2\ \theta\\
&= 2 - \left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{2}{\sqrt{5}}\right)^2\\
&= 2 - \frac{1}{5} - \frac{4}{5}\\
&= \frac{5}{5}\\
&= 1
\end{aligned}$$

10. In the following figure:
$AD \perp BC, AC = 26\ , CD = 10, BC = 42$, $\angle\text{DAC} = \text{x}$ and $\angle\text{B} = \text{y}$.Find the value of :
$$\frac{1}{\sin^2\ y} - \frac{1}{\tan^2\ y}$$



Solution

Given angle $DAC = 90^\circ$ and $\angle ADB = 90^\circ$ in the figure

$$\Rightarrow AC^2 = AD^2 + DC^2 (AC \text{ is hypotenuse in } \triangle ADC)$$
$$\Rightarrow AD^2 = 26^2 - 10^2$$
$$\therefore AD^2 = 576 \text{ and } AD = 24$$

Again

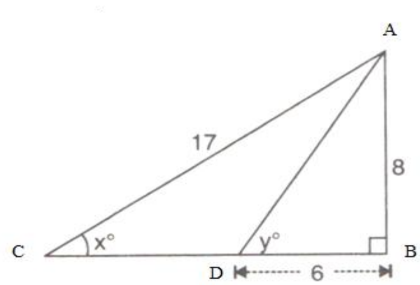
$$\Rightarrow AB^2 = AD^2 + BD^2 (AB\ is \text{ shypotenuse in } \triangle ABD)$$
$$\Rightarrow AB^2 = 24^2 + 32^2$$
$$\therefore AB^2 = 1600 \text{ and } AB = 40$$

$$\sin y = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{24}{40} = \frac{3}{5}$$
$$\tan y = \frac{\text{perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{24}{32} = \frac{3}{4}$$

Therefore,

$$\frac{1}{\sin^2 y} - \frac{1}{\tan^2 y}$$
$$= \frac{1}{\left(\frac{3}{5}\right)^2} - \frac{1}{\left(\frac{3}{4}\right)^2}$$
$$= \frac{25}{9} - \frac{16}{9}$$
$$= \frac{9}{9}$$
$$= 1$$

11. Use the given figure to find : $3\tan x^0 - 2\sin y^0 + 4\cos y^\circ$



Solution

$$\sin\ y^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AD} = \frac{8}{10} = \frac{4}{5}$$
$$\cos\ y^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$
$$\tan\ x^\circ = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{8}{15}$$

Therefore

$$3\tan\ x^\circ - 2\sin\ y^\circ + 4\cos\ y^\circ$$
$$= 3\left(\frac{8}{15}\right) - 2\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right)$$
$$= \frac{8}{5} - \frac{8}{5} + \frac{12}{5}$$
$$= 2\frac{2}{5}$$

12. Given $q\tan\ A = p$, find the value of

$$\frac{p\sin\ A - q\cos\ A}{p\sin\ A + q\cos\ A}$$

Solution

$$q\tan\ A = p$$
$$\tan\ A = \frac{p}{q}$$

Now

$$\frac{p\sin\ A - q\cos\ A}{p\sin\ A + q\cos\ A} = \frac{\frac{p\sin\ A}{\cos\ A} - \frac{q\cos\ A}{\cos\ A}}{\frac{p\sin\ A}{\cos\ A} + \frac{q\cos\ A}{\cos\ A}}$$
$$= \frac{p\tan\ A - q}{p\tan\ A + q}$$
$$= \frac{p\left(\frac{p}{q}\right) - q}{p\left(\frac{p}{q}\right) + q}$$
$$= \frac{\frac{p^2 - q^2}{q}q}{p^2 + q^2}$$
$$= \frac{p^2 - q^2}{p^2 + q^2}$$