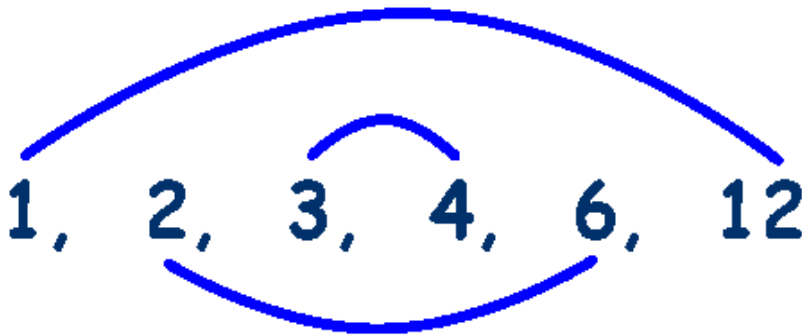


Factors

The numbers which exactly divide a given number are called the Factors of that number.

Factors of 12



We can see, the number 12 is obtained by

1×12 , 2×6 , 3×4 , 4×3 , 6×2 and 12×1

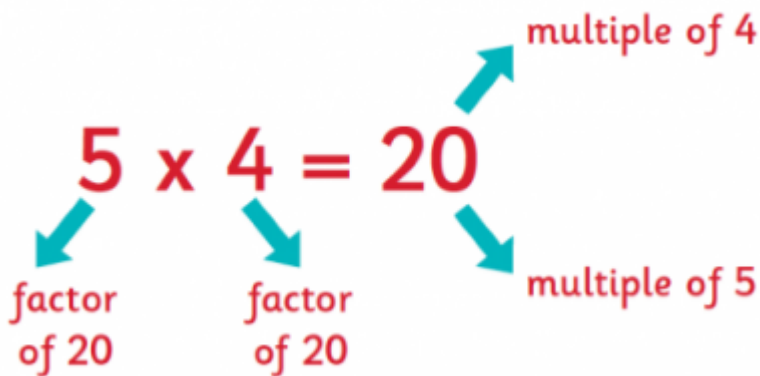
Hence,

1, 2, 3, 4, 6, and 12 are the factors of 12.

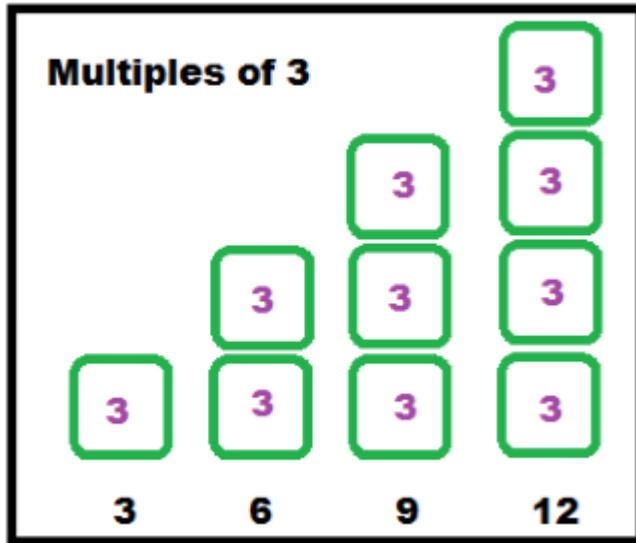
The factors are always less than or equal to the given number.

Multiples

If we say that 4 and 5 are the factors of 20, then 20 is the multiple of both 4 and 5.



List the multiples of 3



Multiples are always more than or equal to the given number.

Some facts about Factors and Multiples

- 1 is the only number that is the factor of every number.
- Every number is the factor of itself.
- All the factors of any number are the exact divisor of that number.
- All the factors are less than or equal to a given number.
- There are limited number of factors of any given number.
- All the multiples of any number are greater than or equal to the given number.
- There are unlimited multiples of any given number.
- Every number is a multiple of itself.

Perfect Number

If the sum of all the factors of any number is double the given number, then that number is called a Perfect Number.

Perfect Number	Factors	Sum of all the factors
6	1, 2, 3, 6	12
28	1, 2, 4, 7, 14, 28	56
496	1, 2, 4, 8, 16, 31, 62, 124, 248, 496	992

Prime Numbers

The numbers whose only factors are 1 and the number itself are called Prime Numbers.

Like 2, 3, 5, 7, 11 etc.

Composite Numbers

All the numbers with more than 2 factors are called composite numbers. In other words, the numbers which are not prime numbers are called Composite Numbers.

Like 4, 6, 8, 10, 12 etc.

Remark: 1 is neither a prime nor a composite number.

Sieve of Eratosthenes Method

This is the method to find all the prime numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1: First of all cross 1, as it is neither prime nor composite.

Step 2: Now mark 2 and cross all the multiples of 2 except 2.

Step 3: Mark 3 and cross all the multiples of 3 except 3.

Step 4: 4 is already crossed so mark 5 and cross all the multiples of 5 except 5.

Step 5: Continue this process until all the numbers are marked square or crossed.

This shows that all the covered numbers are prime numbers and all the crossed numbers are composite numbers except 1.

Even and Odd Numbers

All the multiples of 2 are even numbers. To check whether a number is even or not, we can check the number at one's place. If the number at one's place is 0,2,4,6 and 8 then it is an even number.

The numbers which are not even are called Odd Numbers.

Remark: 2 is the smallest even prime number. All the prime numbers except 2 are odd numbers.

Tests for Divisibility of Numbers

1. Divisibility by 2:

If there are any even numbers i.e. 0, 2, 4, 6, and 8 at the end of the digit then it is divisible by 2.

Example

Check whether the numbers 63 and 240 are divisible by 2 or not.

Solution:

1. The last digit of 63 is 3 i.e. odd number, so 63 is not divisible by 2.
2. The last digit of 240 is 0 i.e. even number, so 240 is divisible by 2.

2. Divisibility by 3:

A given number will only be divisible by 3 if the total of all the digits of that number is a multiple of 3.

Example

Check whether the numbers 623 and 2400 are divisible by 3 or not.

Solution:

1. The sum of the digits of 623 i.e. $6 + 2 + 3 = 11$, is not a multiple of 3, so 623 is not divisible by 3.
2. The sum of the digits of 2400 i.e. $2 + 4 + 0 + 0 = 6$, is a multiple of 3, so 2400 is divisible by 3.

3. Divisibility by 4:

We have to check whether the last two digits of the given number are divisible by 4 or not. If it is divisible by 4 then the number will be divisible by 4.

Example

Check if the numbers 23436 and 2582 are divisible by 4 or not.

Solution:

1. The last two digits of 23436 are 36 which are divisible by 4, so 23436 are divisible by 4.
2. The last two digits of 2582 are 82 which are not divisible by 4, so 2582 is not divisible by 4.

4. Divisibility by 5:

Any given number will be divisible by 5 if the last digit of that number is '0' or '5'.

Example

Check whether the numbers 2348 and 6300 are divisible by 5 or not.

Solution:

1. The last digit of 2348 is 8, so it is not divisible by 5.
2. The last digit of 6300 is 0, so it is divisible by 5.

5. Divisibility by 6:

Any given number will be divisible by 6 if it is divisible by both 2 and 3. So we should do the divisibility test of 2 and 3 with the number and if it is divisible by both, then it is also divisible by 6.

Example

Check if the numbers 342341 and 63000 are divisible by 6 or not.

Solution:

1. 342341 is not divisible by 2 as the digit at one's place is odd and is also not divisible by 3 as the sum of its digits i.e. $3 + 4 + 2 + 3 + 4 + 1 = 17$ is not divisible by 3. Hence 342341 is not divisible by 6.
2. 63000 is divisible by 2 as the digit at ones place is even and is also divisible by 3 as the sum of its digits i.e. $6 + 3 + 0 + 0 + 0 = 9$ is divisible by 3. Hence 63000 is divisible by 6.

6. Divisibility by 7:

Any given number will be divisible by 7 if we double the last digit of the number and then subtract the result from the rest of the digits and check whether the remainder is divisible by 7 or not. If there is a large number of digits then we have to repeat the process until we get the number which could be checked for the divisibility of 7.

Example

Check if the number 2030 is divisible by 7 or not.

Solution:

Given number is 2030

1. Double the last digit, $0 \times 2 = 0$
2. Subtract 0 from the remaining number 203 i.e. $203 - 0 = 203$
3. Double the next digit, $3 \times 2 = 6$
4. Subtract 6 from the remaining number 20 i.e. $20 - 6 = 14$
5. The remainder 14 is divisible by 7 hence the number 203 is divisible by 7.

7. Divisibility by 8:

We have to check whether the last three digits of the given number are divisible by 8 or not. If it is divisible by 8 then the whole number will be divisible by 8.

Example

Check whether the number 74640 is divisible by 8.

Solution:

The last three digits of the number 74640 is 640.

As the number 640 is divisible by 8 hence the number 74640 is also divisible by 8.

8. Divisibility by 9:

Any given number will be divisible by 9 if the sum of all the digits of that number is divisible by 9.

Example

Check whether the numbers 2320 and 6390 are divisible by 9.

Solution:

1. The sum of the digits of 2320 is $2 + 3 + 2 + 0 = 7$ which is not divisible by 9, so 2320 is not divisible by 9.
2. The sum of the digits of 6390 is $6 + 3 + 9 + 0 = 18$ which is divisible by 9, so 6390 is divisible by 9.

9. Divisibility by 10:

Any given number will be divisible by 10 if the last digit of that number is zero.

Example

Check if the numbers 123 and 2630 are divisible by 10 or not.

Solution:

1. The one's place digit is 3 in 123 so it is not divisible by 10.
2. The one's place digit is 0 in 2630 so it is divisible by 10.

Common Factors and Common Multiples

Example: 1

What are the common factors of 25 and 55?

Solution:

Factors of 25 are 1, 5.

Factors of 55 are 1, 5, 11.

Common factors of 25 and 55 are 1 and 5.

Example: 2

Find the common multiples of 3 and 4.

Solution:

Multiples of 3:

0, 3, 6, 9, 12, 15, 18, 21, 24, ...

Multiples of 4:

0, 4, 8, 12, 16, 20, 24, 28, ...

Common multiples of 3 and 4 are 0, 12, 24, and so on.

Co-prime Numbers

If 1 is the only common factor between two numbers, then they are said to be Co-prime Numbers.

Example

Check whether 7 and 15 are co-prime numbers or not.

Solution:

Factors of 7 are 1 and 7.

Factors of 15 are 1, 3, 5, and 15.

The common factor of 7 and 15 is 1 only. Hence they are the co-prime numbers.

Some more Divisibility Rules

1. Let a and b are two given numbers. If a is divisible by b then it will also be divisible by all the factors of b.

If 24 is divisible by 12 then 24 will also be divisible by all the factors of 12 (i.e. 2, 3, 4, 6).

2. Let a and b are two co-prime numbers. If c is divisible by a and b then c will also be divisible by the product of a and b (ab).

If 24 is divisible by 2 and 3 which are the co-prime numbers then 24 will also be divisible by the product of 2 and 3 ($2 \times 3 = 6$).

3. If a and b are divisible by c , then $a + b$ will also be divisible by c .

If 24 and 12 are divisible by 4, then $24 + 12 = 36$ will also be divisible by 4.

4. If a and b are divisible by c , then $a - b$ will also be divisible by c .

If 24 and 12 are divisible by 4, then $24 - 12 = 12$ will also be divisible by 4.

Prime Factorisation

Prime Factorisation is the process of finding all the prime factors of a number.

There are two methods to find the prime factors of a number-

1. Prime factorisation using a factor tree

We can find the prime factors of 70 in two ways.



The prime factors of 70 are 2, 5, and 7 in both cases.

2. Repeated Division Method

Find the prime factorisation of 64 and 80.

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

The prime factorisation of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

The prime factorisation of $80 = 2 \times 2 \times 2 \times 2 \times 5$.

Highest Common Factor (HCF)

The highest common factor (HCF) of two or more given numbers is the greatest of their common factors.

Its other name is (GCD) Greatest Common Divisor.

Method to find HCF

To find the HCF of given numbers, we have to find the prime factorisation of each number and then find the HCF.

Example

Find the HCF of 60 and 72.

Solution:

First, we have to find the prime factorization of 60 and 72.

Then encircle the common factors.

$$\begin{array}{l} 60 = 2 \times 2 \times 3 \times 5 \\ 72 = 2 \times 2 \times 2 \times 3 \times 3 \end{array}$$

HCF of 60 and 72 is $2 \times 2 \times 3 = 12$.

Lowest Common Multiple (LCM)

The lowest common multiple of two or more given numbers is the smallest of their common multiples.

Methods to find LCM

1. Prime Factorisation Method

To find LCM we have to find the prime factorisation of all the given numbers and then multiply all the prime factors which have occurred the maximum number of times.

Example

Find the LCM of 60 and 72.

Solution:

First, we have to find the prime factorisation of 60 and 72.

Then encircle the common factors.

$$\begin{aligned} 60 &= 2 \times 2 \times 3 \times 5 \\ 72 &= 2 \times 2 \times 2 \times 3 \times 3 \end{aligned}$$

To find the LCM, we will count the common factors once and then multiply them with the other remaining factors.

LCM of 60 and 72 is $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

2. Repeated Division Method

If we have to find the LCM of so many numbers then we use this method.

Example

Find the LCM of 105, 216, and 314.

Solution:

Use the repeated division method on all the numbers together and divide until we get 1 in the last row.

2	105	216	314
2	105	108	157
2	105	54	157
3	105	27	157
3	35	9	157
3	35	3	157
5	35	1	157
7	7	1	157
157	1	1	157
	1	1	1

LCM of 105, 216, and 314 is $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 \times 157 = 1186920$

Real-life problems related to HCF and LCM

Example: 1

Two containers have 240 liters and 1024 liters of petrol respectively. Calculate the maximum capacity of a container that can measure the petrol of both the containers when used an exact number of times.

Solution:

As we have to find the capacity of the container which is the exact divisor of the capacities of both the containers, i. e. maximum capacity, so we need to calculate the HCF.

2	240
2	120
2	60
2	30
3	15
5	5
	1

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

The common factors of 240 and 1024 are $2 \times 2 \times 2 \times 2$. Thus, the HCF of 240 and 1024 is 16. Therefore, the maximum capacity of the required container is 16 liters.

Example: 2

What could be the least number which when we divided by 20, 25, and 30 leaves a remainder of 6 in every case?

Solution:

As we have to find the least number so we will calculate the LCM first.

2	20	25	30
2	10	25	15
3	5	25	15
5	5	25	5
5	1	5	1
	1	1	1

So, $LCM = 2 \times 2 \times 3 \times 5 \times 5$.

LCM of 20, 25, and 30 is $2 \times 2 \times 3 \times 5 \times 5 = 300$.

Here 300 is the least number which when divided by 20, 25, and 30 leave a remainder 0 in each case. But we have to find the least number which leaves the remainder 6 in all cases. Hence, the required number is 6 more than 300.

The required least number = $300 + 6 = 306$.