Sample Question Paper (TERM - I)

Solutions

Section - A

Solution 1: Option (D) is correct.

For a 2×2 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Given that

$$a_{ij} = \begin{cases} 1, & i \neq j \\ 0, & i = j \end{cases}$$

Thus,

$$a_{11} = 0,\ a_{22} = 0,\ a_{12} = 1,\ a_{21} = 1$$

So, our matrix becomes

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now,

$$\mathbf{A}^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0(0) + 1(1) & 0(1) + 1(0) \\ 1(0) + 0(1) & 1(1) + 0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solution 2: Option (C) is correct.

Since,$f(x) = \frac{kx^3}{3}$

Now, differentiate then we get,

$$f'(x) = \frac{3kx^2}{3}$$

put $x = 3$ then we get,

$$f'(3) = \frac{k(3)^3}{3}$$

$$4 = \frac{k(27)}{3}$$

$$k = \frac{12}{27} = \frac{4}{9}$$

Solution 3: Option (C) is correct.

Explanation:

$$\begin{vmatrix} \sin\frac{\pi}{2} & 1 \\ \left(\sin\frac{\pi}{4}\right)^2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ \frac{1}{2} & 2 \end{vmatrix} = 2 - \frac{1}{2} = \frac{3}{2}$$

Solution 4: Option (C) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping (bijective mappings) from A to B is

$n!$ if $m = n$

0, if $m \neq n$

Given that, $m = 5$ and $n = 6 \Rightarrow m \neq n$ Number of one-one and onto mapping $= 0$

Solution 5: Option (B) is correct.

Explanation: $A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A.\,A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha \cdot \cos\alpha + \sin\alpha \cdot \sin\alpha & -\cos\alpha \cdot \sin\alpha + \sin\alpha \cdot \cos\alpha \\ -\sin\alpha \cdot \cos\alpha + \sin\alpha \cdot \cos\alpha & \sin\alpha \cdot \sin\alpha + \cos\alpha \cdot \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

{Since, Since, $\cos^2\alpha + \sin^2\alpha = 1$}

Solution 6: Option (A) is correct.

Explanation: $f: R \rightarrow R$ is defined as $f(x) = 5x$.

Let $x, y \in R$ such that $f(x) = f(y) \Rightarrow 5x = 5y \Rightarrow x = y$

Therefore, f is one-one. Also, for any real number (y) in co-domain R, there exists y/5 in R such that

$$f\left(\frac{y}{5}\right) = 5f\left(\frac{y}{5}\right) = y$$

Therefore, f is onto. Hence, function f is one-one and onto.

Solution 7: Option (A) is correct.

Explanation: $y = \text{cosec}x \times \log x$

$$\frac{dy}{dx} = \frac{\text{cosec}x}{x} + \log x \times (-\text{cosec}x \times \cot x)$$

$$\frac{dy}{dx} = -\text{cosec}\,x \times \cot x \times \log x + \frac{\text{cosec}x}{x}$$

Solution 8: Option (D) is correct.

Explanation: In mathematics, particularly in linear algebra, a skew symmetric (or anti symmetric or antisymmetric) matrix is a square matrix whose transpose equals its negative; that is, it satisfies the condition $A = -A^t$.

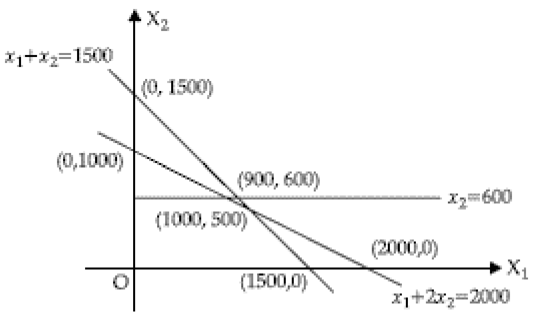
Solution 9: Option (A) is correct.

Explanation:

$$\begin{aligned} y &= e^{-\sin^{-1}}x \\ \frac{dy}{dx} &= e^{-\sin^{-1}}x \times \frac{d}{dx}(-\sin^{-1} x) \\ &= y \cdot \frac{-1}{\sqrt{1-x^2}} = \frac{-y}{\sqrt{1-x^2}} \end{aligned}$$

Solution 10: Option (D) is correct.

Explanation:



From the graph, it is clear that the point (2000,0) is outside.

Solution 11: Option (A) is correct

Given

$R = \{(a, b): |a - b| \text{ is a multiple of } 4\}$

$\&A = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$

Here $a = 1$,

We need to find values of b such that $(a, b) \in R$

a	b	|a − b|	Is |a − b| a multiple of 4
1	1	|1−1| =|0| =0	Yes
1	2	|1−2| =|−1| = 1	No
1	3	|1−3| =|−2| = 2	No
1	5	|1−5| =|−4| = 4	Yes
1	9	|1−9| =|−8| = 8	Yes

The set of elements related to 1 are $\{1,5,9\}$

Solution 12: Option (A) is correct.

Explanation: Tangent and normal perpendicular to each other.

$y = 3x^2 - 7x + 5$

$\frac{dy}{dx} = 6x - 7$

$\left.\frac{dy}{dx}\right]_{(0,5)} = -7$

$\therefore$ Slope of normal $= \frac{1}{7}$

Equation of normal is

$\frac{(y-5)}{(x-0)} = \frac{1}{7}$

$\Rightarrow 7y - 35 = x$

$\Rightarrow x - 7y + 35 = 0$

Solution 13: Option (C) is correct.

Explanation: $|x|$ is not differentiable at $x = 0$

Solution 14: Option (C) is correct.

Since A is singular matrix

Determinant of $A = |A| = 0$

$|A| = \begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix}$

$0 = k(2k) - 4 \times 8$

$0 = 2k^2 - 32$

$32 = 2k^2$

$2k^2 = 32$

$k^2 = 16$

$k = \pm 4$

Solution 15: Option (D) is correct.

Explanation:

$y = \log(\cos e^x)$

$\frac{dy}{dx} = \frac{1}{\cos x} \times \frac{d}{dx}(\cos e^x) = \frac{1}{\cos e^x} \times (-\sin e^x)e^x = -e^x \tan e^x$

Solution 16: Option (C) is correct.

Explanation: let $f(x) = 5x^2 - 32x$

$f'(x) = 10x - 32$

$10x - 32 = 0$

$f''(x)$ for $x = 3.2$ and $f'(x) < 0$ for $x > 3.2$

Solution 17: Option (A) is correct.

Explanation: $f(x) = x + \frac{1}{x}, x > 0$

$$\Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}, x > 0$$

As normal to $f(x)$ is $\perp$ to given line

$$\Rightarrow \left(\frac{x^2}{1 - x^2}\right) \times \frac{3}{4} = -1 (m_1 m_2 = -1)$$

$$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

But $x > 0, \therefore x = 2$

Therefore point $= \left(2, \frac{5}{2}\right)$

Solution 18: Option (A) is correct.

Explanation: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Value of $A^2 - 5A + 7I_2$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, all the elements of matrix are zero. So, given matrix is null/zero matrix.

Solution 19: Option (D) is correct.

Explanation: The given matrix is not a skew symmetric matrix as $A' \neq -A$. By Definition; we know, A matrix is a skew- symmetric matrix if $A' = -A$.

Solution 20: Option (A) is correct.

Explanation:

$$y = \log\sqrt{1-x^2} = \frac{1}{2}\log(1-x^2)$$
$$\frac{dy}{dx} = \frac{1}{2(1-x^2)} \times (-2x) = \frac{-x}{1-x^2}$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$
$$= \frac{(1-x^2)(-1)-(-x)(-2x)}{(1-x^2)^2}$$
$$= \frac{-1+x^2-2x^2}{(1-x^2)^2} = \frac{-1-x^2}{(1-x^2)^2}$$

Section - B

Solution 21: Option (A) is correct.

Explanation:

$x = a\sec\theta$

$\Rightarrow \frac{dx}{d\theta} = a\tan\theta\sec\theta$

$y = b\tan\theta$

$\Rightarrow \frac{dy}{d\theta} = b\sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{b}{a}\operatorname{cosec}\theta$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\operatorname{cosec}\theta \cdot \cot\theta \cdot \frac{d\theta}{dx} = \frac{-b}{a^2}\cot^3\theta$

$\therefore \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2}$

Solution 22: Option (C) is correct.

Explanation: Z is minimum −24 at (0,8)

Solution 23: Option (D) is correct.

Explanation:

Corner points of feasible region	$Z = 30x + 50y$
(5,0)	150
(9,0)	270
(0,3)	150
(0,6)	300

Minimum value of Z occurs at two points

Solution 24: Option (B) is correct.

Explanation:

By definition, a relation in Z is said to be reflexive if xRx, $\forall x \in Z$. So,

$a - a = 0 \Rightarrow 3$ divides $a - a \Rightarrow aRa$. Hence R is reflexive.

Solution 25: Option (A) is correct.

Explanation:

Given function is continuous but not differentiable at $x = 0$

Solution 26: Option (A) is correct.

Explanation:

Let Food X be p and Food Y be q

Formulate the constraints as per statement.

The Constraints are : $p + 2q \geq 15; 3p + 2q \geq 17; p + q \leq 6; p \geq 0; q \geq 0$

And objective function is $Z = 16p + 20q$

By solving the above equations, Corner points will be (1,7), (1,5), (3,4), (5,1) and (0,6)

After putting these points in Z, we get the least cost of the mixture, i.e., 100 .

Solution 27: Option (B) is correct.

Explanation:

For bijection on Z, f(x) must be one-one and onto. Function $f(x) = x^2 + 7$ is many-one as $f(1) = f(-1)$ Range of $f(x) = x^3$ is not Z for $x \in Z$. Also $f(x) = 4x + 1$ takes only values of type $= 4k + 1$ for $x \in k \in Z$ But $f(x) = x + 8$ takes all integral values for $x \in Z$.

Hence $f(x) = x + 8$ is a bijection of Z.

Solution 28: Option (A) is correct.

Explanation:

By definition, a relation in A is said to be reflexive if xRx, $\forall x \in A$. So R is true.

The number of reflexive relations on a set containing n elements is 2^{n^2-n} .

Here $n = 4$.

The number of reflexive relations on a set $A = 2^{12}$

Solution 29: Option (B) is correct.

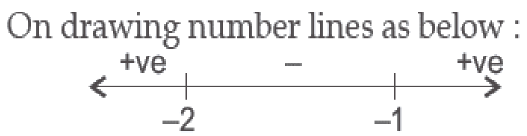
Explanation:

Given that,

$$\begin{aligned} f(x) &= 2x^3 + 9x^2 + 12x - 1 \\ f'(x) &= 6x^2 + 18x + 12 \\ &= 6(x^2 + 3x + 2) \\ &= 6(x+2)(x+1) \end{aligned}$$

So, $f'(x) \leq 0$, for decreasing.

On drawing number line as below:



We see that $f'(x)$ is decreasing in $(-2, -1)$

Solution 30: Option (B) is correct.

Explanation:

Corner Points	$\mathbf{Z} = 7x + y$
$(0, 3)$	3
$\left(\frac{1}{2}, \frac{5}{2}\right)$	6
$(0,5)$	5

Solution 31: Option (A) is correct.

Explanation:

Given that,A and B are symmetric matrices.

$\Rightarrow$ $A = A'$ and $B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$ 1

$\Rightarrow$ $(AB - BA)' = B'A' - A'B'$

[By reversal law]

$\Rightarrow$ $(AB - BA)' = BA - AB$ [From Eq. (1)] $\Rightarrow$ $(AB - BA)' = -(AB - BA)$

$\Rightarrow (AB - BA)$ is a skew-symmetric matrix.

Solution 32: Option (C) is correct.

Explanation:

We know that, in a square matrix, if $b_{ij} = 0$ when $i \neq j$ then it is said to be a diagonal matrix. Here, $b_{12}, b_{13} \dots \neq 0$ so the given matrix is not a diagonal matrix.

Now,

$$B = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix} = -B$$

So, the given matrix is a skew-symmetrio matrix, since we know that in a square matrix B, if $B' = -B$, then it is called skew-symmetric matrix

Solution 33: Option (A) is correct.

Explanation:

Given, $A = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix},$$

$X = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$,

$$Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{aligned} AB + XY &= \begin{bmatrix} 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \\ &= [6 - 6 + 8] + [2 + 6 + 12] \\ &= [8] + [20] \\ &= [28] \end{aligned}$$

Solution 34: Option (A) is correct.

Explanation:

Given that the equation of curve is

$y(1 + x^2) = 2 - x \quad \dots\dots 1$

On differentiating with respect to x, we get

$$\therefore\ y(0+2x)+(1+x^2)\cdot\frac{dy}{dx}=0-1$$

$$\Rightarrow\ 2xy+(1+x^2)\frac{dy}{dx}=-1$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{-1-2xy}{1+x^2}\qquad .....2$$

Since, the given curve passes through x-axis, i.e.,

$y=0$

$0(1+x^2)=2-x$ [By using Eq. (1)]

$\therefore\ x=2$

So the curve passes through the point (2,0).

$$\therefore\ \left(\frac{dy}{dx}\right)_{(2,0)}=\frac{-1-2\times 0}{1+2^2}=-\frac{1}{5}=\text{ Slope of the curve}$$

$\therefore$ Slope of tangent to the curve $=-\frac{1}{5}$

$\therefore$ Equation of tangent to the curve passsing through (2,0) is

$$y-0=-\frac{1}{5}(x-2)$$

$$\Rightarrow\ y+\frac{x}{5}=+\frac{2}{5}$$

$$\Rightarrow 5y+x=2$$

Solution 35: Option (C) is correct.

Explanation:

$x-y$ is an integer

$x-x=0$ is an integer

$\Rightarrow$ A is reflexive

$x-y$ is an integer

$\Rightarrow y-x$ is an integer

$\Rightarrow$ A is symmetric

Now, $x-y, y-z$ are integers

We know, as sum of integers is also an integer

$\Rightarrow (x-y)+(y-z)=x-z$ is an integer $\Rightarrow$ A is transitive

Solution 36: Option (B) is correct.

Explanation:

aRb $\Rightarrow$ a is brother of b.

This does not mean b is also a brother of a as b can be sister of a.

Hence, R is not symmetric.

aRb $\Rightarrow$ a is brother of b

and bRc $\Rightarrow$ b is a brother of c.

So, a is brother of c.

Hence, R is transitive.

Solution 37: Option (D) is correct.

Explanation:

$y = e^x \log \sin x$

$$\frac{dy}{dx} = \frac{e^x}{\sin x}\cos x + e^x \log \sin x$$

$$= e^x \cos x + e^x \log \sin x$$

$$\frac{d^2y}{dx^2} = -e^x \operatorname{cosec}^2 x + e^x \cos x + \frac{e^x}{\sin x}\cos x + e^x \log \sin x$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-\operatorname{cosec}^2 x + \cot x)$$

Solution 38: Option (B) is correct.

Explanation:

Function has critical points $x = \frac{\pi}{2}, \frac{3\pi}{2}$. At critical points, The sign of $f'(x)$ changes,

So the function increases and then decreases in the given interval.

Solution 39: Option (C) is correct.

Explanation:

Given,

$$y = \begin{cases} k\cos x, x < \frac{\pi}{4} \\ m\sin x, x > \frac{\pi}{4} \\ y = 3, x = \frac{\pi}{4} \end{cases}$$

$$\text{L.H.L} = \lim_{x \to \frac{\pi}{4}^-} k\cos x$$

$$= \lim_{h \to 0} k\cos\left(\frac{\pi}{4} - h\right) = k \times \frac{1}{\sqrt{2}}$$

$$f\left(\frac{\pi}{4}\right) = \text{LHL}$$

$$3 = \frac{k}{\sqrt{2}}$$

$$k = 3\sqrt{2}$$

Given, $f\left(\frac{\pi}{4}\right) = \text{RHL} = \lim_{x \to \frac{\pi}{4}^+} m\sin x$

$3 = m \times \frac{1}{\sqrt{2}}$

$3 = \frac{m}{\sqrt{2}}$

$m = 3\sqrt{2}$

$k + m = 6\sqrt{2}$

Solution 40: Option (C) is correct.

Explanation:

$\frac{dy}{dx} = 5 - 6x^2$

$m = 5 - 6x^2$

Now, $\frac{dm}{dt} = -12x\frac{dx}{dt} = -24x$ $\left(\because \frac{dx}{dt} = 2\text{unit/sec}\right)$, $\therefore \left(\frac{dm}{dt}\right)_{\text{at } x=3} = -72$

Section - C

Solution 41: Option (D) is correct.

Explanation:

Corner points	Value of Z
(0,0)	0(min.)
(0,4)	8
(3,1)	11(max.)
(2,0)	6

Solution 42: Option (C) is correct.

Explanation: Slope of the tangent

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = \frac{1 - \cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2} - 1$$

Solution 43: Option (D) is correct.

Explanation : Let $f(x) = (x-1)^2 + 3$

Then, $f'(x) = 2(x-1)$

For critical points, put $f'(x) = 0$, $\therefore x = 1$

Now ,

$f(-3) = (-3-1)^2 + 3 = 16 + 3 = 19$

$f(1) = 3$

$\therefore$ Maximum value $= 19$

Solution 44: Option (C) is correct.

Explanation :

Corner points	Value of Z
(0,5)	2500
(4,3)	2300 (min.)
(0,6)	3000

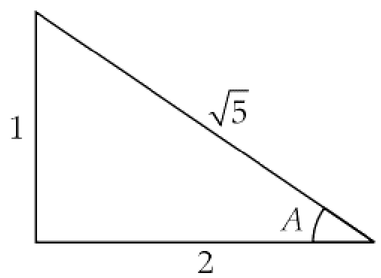
Solution 45: Option (D) is correct.

Explanation: $|A| = \begin{vmatrix} 2 & -1 & -1 \\ 3 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 2(-1+1) + 1(-3+1) - 1(3-1) = -4$

Solution 46: Option (C) is correct.

Explanation:

$A = \tan^{-1}\frac{1}{2}$



$\Rightarrow \tan A = \frac{1}{2}$

$A = \tan^{-1}\frac{1}{2}$

$\Rightarrow\ \tan A = \frac{1}{2} \Rightarrow\ \sin A = \frac{1}{\sqrt{5}}$

Solution 47: Option (C) is correct.

Explanation: Since ABC is a triangle,

$\therefore\ A + B + C = 180^\circ$

$\cos(A + B + C)\ = \cos 180^\circ = -1$

Solution 48: Option (B) is correct.

Explanation:

Given,

$B = \tan^{-1}\frac{1}{3}$

$\Rightarrow\ \tan B = \frac{1}{2}$

$\therefore\ \cos B = \frac{3}{\sqrt{10}}$

$B = \cos^{-1}\frac{3}{\sqrt{10}}$

$\Rightarrow\ x = \frac{3}{\sqrt{10}}$

Solution 49: Option (A) is correct.

Explanation:

$A = \tan^{-1}\frac{1}{2}$

$\Rightarrow\ \tan A = \frac{1}{2}$

$\therefore\ \sin A = \frac{1}{\sqrt{5}}$

$A = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \Rightarrow\ x = \frac{1}{\sqrt{5}}$

Solution 50: Option (D) is correct.

Explanation:

$$\angle C = \pi - (A + B) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$