Board – CBSE | Class – 8th | Topic – Direct and Inverse Proportion

- **DIRECT VARIATION**
  - Consider the following table which shows various numbers of books (each of the same cost) denoted by x and the corresponding cost denoted by y.

x (No. of Books)	2	3	5	10	15
y (Cost in Rupees)	15	75	125	250	375

  - Here, we note that there is an increase in cost corresponding to the increase in the number of books. Hence, it is a case of direct variation.
  - In this case, if we compare the ratio of the different number of books to the corresponding costs, then we have:

$$\frac{2}{50},\quad \frac{3}{75},\quad \frac{5}{125},\quad \frac{10}{250},\quad \frac{15}{375},$$

$$\downarrow\quad\ \downarrow\quad\ \downarrow\quad\ \downarrow\quad\ \downarrow$$

$$\text{or}\quad \frac{1}{25},\quad \frac{1}{25},\quad \frac{1}{25},\quad \frac{1}{25},\quad \frac{1}{25},$$

Thus is, each ratio reduces to $\frac{1}{25}$ which is constant. We may express is in a general form as:

$\frac{x}{y} = k(\text{ constant })$

Thus, we conclude that,

When two quantities x and y vary such that the ratio $\frac{x}{y}$ remains constant and positive, then we say that x and y vary directly and the variation is called a Direct Variation.

In Mathematical language, it may be written as,

$\frac{x}{y} = k \text{ or } x = ky$

Let us consider any two values of x, say x_1 and x_2 with their corresponding values of y as y_1 and y_2. We have and $x_1 = ky_1$

$x_2 = ky_2$

$\therefore \frac{x_1}{x_2} = \frac{ky_1}{ky_2}$

Or $\frac{x_1}{x_2} = \frac{y_1}{y_2}$, which helps us to find the value of any one of x_1, x_2, y_1, and y_2, when the other three variables are known.

- **INVERSE VARIATION**
  - Consider the following table showing the various numbers of men and the corresponding number of days to complete the work.

x (No. of men)	40	20	10	8	5	1
y (No. of days)	1	2	4	5	8	40

Here, the number of men is denoted by x and the corresponding number of days by y. In this case, when the number of men increases, the corresponding number of days decreases. But, by a careful observation, we find that the product of the corresponding number of men and days is always the same:

$40 \times 1 = 40$

$20 \times 2 = 40$

$10 \times 4 = 40$

$8 \times 5 = 40$

$5 \times 8 = 40$

$1 \times 40 = 40$

That is the product (40) is constant.

In general, it may be expressed as $xy = k$(constant)

Let x_1 and x_2 be two values of x and their corresponding values of y be y_1 and y_2.

Then, $x_1y_1 = k$ and $x_2y_2 = k$

$\therefore \frac{x_1y_1}{x_2y_2} = \frac{k}{k} = 1$

or $x_1y_1 = x_2y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

Hence, we conclude that, if two quantities x and y vary such that their product xy remains constant, then we say that x and y vary inversely and the variation is called inverse variation.

The relation $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ is used to find the value of any one of x_1, x_2, y_1, and y_2 if the other three variables are known.

- **TIME AND WORK**
  - We use the principles of direct and indirect variations to solve problems on 'time and work', such as:

    "More men do more work and lesser men do less work" (Direct variation)

    "More men take less time to do a work and lesser men take more time to do the same work." (Indirect variation)

    The problems on "time and work" are divided into two categories:

    (i) To find the work done in a given period of time.

    (ii) To find the time required to complete a given job.

  ❖ **Working Rules**

    We shall use the unitary method by considering the following fundamental rules for solving problems regarding time and work:

    (i) A complete job or work is taken to be one.

    (ii) Time to complete a work $= \frac{\text{Total work to be done}}{\text{Part of the work done in one day}}$.

- **TIME, DISTANCE, AND SPEED**
  - We generally say that a body is covering so many kilometers every hour or so many meters in every second. We define the speed of a body as the distance covered in unit time. Here, unit time can be one hour or one minute or one second and a body means an object.
  - Thus, speed is expressed in meters per second(m/s) or kilometers per second (km/s) or centimeters per second (cm/s). To find the speed of a moving object, we divide the distance covered by the time taken.

    $\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}}$ or Time $= \frac{\text{Distance}}{\text{Speed}}$ or Distance $=$ Speed $\times$ Time.