Sample Question Paper (TERM - I)

Solutions

Section - A

Ans. 1	(D) **Explanation:** The cosine function is periodic so to calculate its inverse function we need to make the function bijective. For that we have to consider an interval in which all values of the function exist and do not repeat. Now for the inverse of a function the domain becomes range and the range becomes domain. Thus the range of cosine function, that is, $[-1,1]$ becomes the domain of inverse function.
Ans. 2	(A) **Explanation**: $x = a\cos^2\theta, y = b\sin^2\theta$ $\frac{dx}{d\theta} = 2a\cos\theta(-\sin\theta), \frac{dy}{dx} = 2b\sin\theta\cos\theta$ $\frac{dx}{d\theta} = -a\sin 2\theta, \frac{dy}{d\theta} = b\sin 2\theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\sin 2\theta}{-a\sin 2\theta} = -\frac{b}{a}$
Ans. 3	(B) **Explanation:** $y = 3x^2 - x^3$ Slope $= \frac{dy}{dx} = 6x - 3x^2$ $\left.\frac{dy}{dx}\right]_{x=2} = 12 - 12 = 0$ Slope $= \tan\theta$ $0 = \tan\theta$ $\tan 0^\circ = \tan\theta$ $\theta = 0^\circ$

Ans. 4	(C) Explanation: Since corresponding entries of equal matrices are equal, So $x = 3$ $3x - y = 2$ $2x + z = 4$ $3y - w = 7$ Put the value of $x = 3$ from equation 1 in equation 2 $3x - y = 2$ $3(3) - y = 2$ $9 - y = 2$ $y = 9 - 2$ $y = 7$
Ans. 5	(B) **Explanation:** $y = x^6 + \log x^2$ $\frac{dy}{dx} = 6x^5 + \frac{1}{x^2} \times 2x = 6x^5 + \frac{2}{x}$
Ans. 6	(D) **Explanation:** $\text{adj } A = \begin{bmatrix} -1 & -5 \\ -6 & 3 \end{bmatrix}$ $A(\text{adj } A) = \begin{bmatrix} 3 & 5 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} -1 & -5 \\ -6 & -1 \end{bmatrix}$ $= \begin{bmatrix} -3 - 30 & -15 - 5 \\ -6 + 6 & -30 + 1 \end{bmatrix}$ $= \begin{bmatrix} -33 & -20 \\ 0 & -29 \end{bmatrix}$ $\therefore \lvert A(\text{adj } A) \rvert = -33 \times -29 - 0$ $= 957$
Ans. 7	(B) **Explanation:** On applying Second Derivative test, At $x = 1.5$ $f''(x) > 0$

Ans. 8	(B) **Explanation:** The equation of the given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$ On differentiating both sides with respect to x, we have: $\frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0$ $\Rightarrow \frac{dy}{dx}=\frac{-16x}{9_y}$ The tangent is parallel to the y-axis if the slope of the normal is 0 , which gives $\frac{-1}{\left(\frac{-16x}{9y}\right)}=\frac{9y}{16x}=0 \Rightarrow y=0$ From , $\frac{x^2}{9}+\frac{y^2}{16}=1$, for $y=0 \Rightarrow x=\pm 3$
Ans. 9	(B) **Explanation:** Vertices of feasible region are: $A(0,20), B(8,12), C(16,0) \& O(0,0)$ $Z(A)=360, Z(B)=392, Z(C)=352$ $\text{Max } Z=392$
Ans. 10	(A) **Explanation:** First derivative is always positive in the given interval.
Ans. 11	(A) **Explanation:** Tangent and normal are perpendicular to each other. $y=3x^2-7x+5$ $\frac{dy}{dx}=6x-7$ $\left.\frac{dy}{dx}\right]_{(0,5)}=-7$ $\therefore$ Slope of normal $=\frac{1}{7}$

	Equation of normal is $\frac{(y-5)}{(x-0)} = \frac{1}{7}$ $\Rightarrow\ 7y - 35 = x \Rightarrow\ x - 7y + 35 = 0$
Ans. 12	(C) **Explanation:** Take $b = 8$, $\Rightarrow a = 6.$ Hence, $(6,8) \in R$
Ans. 13	(B) **Explanation:** For the inverse of a function to exist the function should be bijective which none of the trigonometric function is as they are periodic functions.
Ans. 14	(A) **Explanation:** $A = [2 \quad 3 \quad 5]$ and $A' = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ $\therefore\ A.\ A' = [2 \quad 3 \quad 5] \cdot \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = 4 + 9 + 25 = 38$
Ans. 15	(C) **Explanation:** let $f(x) = 5x^2 - 32x$ $f'(x) = 10x - 32$ $10x - 32 = 0$ $x = 3.2$ $f''(x) > 0$
Ans. 16	(B) **Explanation:** The sum of the product of co-factors of a row and the elements of another row is always 0.

Ans. 17	(B) **Explanation:** The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ means that we need to find an angle in the principaal branch of the function where the sine function is equal to $\frac{1}{2}$. Hence the required value is $\frac{\pi}{6}$,
Ans. 18	(A) Explanation : $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \Rightarrow A^2 = A \cdot A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ $= \begin{bmatrix} 4+4+4 & 4+4+4 & 4+4+4 \\ 4+4+4 & 4+4+4 & 4+4+4 \\ 4+4+4 & 4+4+4 & 4+4+4 \end{bmatrix} = \begin{bmatrix} 12 & 12 & 12 \\ 12 & 12 & 12 \\ 12 & 12 & 12 \end{bmatrix}$ $\Rightarrow A^3 = A^2 \cdot A = \begin{bmatrix} 12 & 12 & 12 \\ 12 & 12 & 12 \\ 12 & 12 & 12 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 72 & 72 & 72 \\ 72 & 72 & 72 \\ 72 & 72 & 72 \end{bmatrix}$ $\Rightarrow 35\,A = \begin{bmatrix} 70 & 70 & 70 \\ 70 & 70 & 70 \\ 70 & 70 & 70 \end{bmatrix}$ $A^2 - 35\,A = \begin{bmatrix} 72 & 72 & 72 \\ 72 & 72 & 72 \\ 72 & 72 & 72 \end{bmatrix} - \begin{bmatrix} 70 & 70 & 70 \\ 70 & 70 & 70 \\ 70 & 70 & 70 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} = A$
Ans. 19	(B) **Explanation:** Let R be the relation in the set $\{p, q, r\}$ is given by: $R = \{(p, p), (q, q), (r, r), (p, q)\}$ (a) $\{(p, p), (q, q), (r, r)\} \in R$ Therefore, R is transitive and reflexive. (b) $(p, q) \in R$ but $(q, p) \notin R$ Therefore, R is not symmetric.
Ans. 20	(C) **Explanation:** Given $A^2 = A$ $\therefore\ (I + A)^2 - 3A = I^2 + 2IA + A^2 - 3A = I + 2A + A - 3A = I$

	Section - B
Ans. 21	(A) **Explanation:** All the elements in the domain have a unique value in the range. Also, the codomain of the function is equal to its range.
Ans. 22	(B) **Explanation:** $y^x + x^y = 0$ or $e^{x\log y} + e^{y\log x} = 0$ Differentiating both sides w.r.t. x. $y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] + x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right] = 0$ $\left[y^x \times \frac{x}{y} + x^y \times \log x\right]\frac{dy}{dx} = -[y^x\log y + x^{y-1} \cdot y]$ $\frac{dy}{dx} = \frac{-[y^x\log y + yx^{y-1}]}{[xy^{x-1} + x^y\log x]}$
Ans. 23	(A) **Explanation :**

Corner points	Corresponding value of $F = 6x + 4y$
(0,2)	8 ← Minimum
(3,0)	18
(6,0)	36
(6,8)	68 ← Maximum
(0,5)	20

Hence, minimum value of F occurs at the point (0,2)

Ans. 24	(C) **Explanation:** $y = \frac{2x^2}{3}$ Slope of the curve $= \frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x^2}{3}\right) = \frac{2\times2x}{3} = \frac{4x}{3}$ at $x = -1$ $\frac{dy}{dx} = \frac{-4}{3}$ Parallel lines have same slopes $\therefore$ Slope of tangent $= \frac{-4}{3}$
Ans. 25	(B) **Explanation:** For bijection on Z, f(x) must be one-one and onto. Function $f(x) = x^2 + 9$ is many-one as $f(1) = f(-1)$ Range of $f(x) = x^5$ is not Z for $x \in Z$. Also $f(x) = 6x + 5$ takes only values of type $= 6k + 5$ for $x \in k \in Z$ But $f(x) = x + 7$ takes all integral values for $x \in Z$. Hence $f(x) = x + 7$ is a bijection of Z.
Ans. 26	(A) **Explanation:** Given that,$y = \log\left(\frac{1+x^3}{1-x^3}\right)$ $y = \log(1 + x^3) - \log(1 - x^3)$ Differentiate with respect to x, we have $\frac{dy}{dx} = \frac{d}{dx}[\log(1 + x^3)] - \frac{d}{dx}[\log(1 - x^3)]$ $= \frac{3x^2}{1 + x^3} - \frac{-3x^2}{1 - x^3} = 3x^2\left(\frac{2}{(1 - x^3)(1 + x^3)}\right) = \frac{6x^2}{1 - x^6}$
Ans. 27	(B) **Explanation:** A set of values of the given variables satisfying the constraints of a L.P.P. is called a solution of L.P.P.

Ans. 28	(C) Explanation: $\frac{dy}{dx} = 3\cos x - 2\sin x$ $\frac{d^2y}{dx^2} = -(3\sin x + 2\cos x) = -y$ Then $y + \frac{d^2y}{dx^2} = y - y = 0$
Ans. 29	(D) **Explanation :** $y = e^x \log \sin x$ $\frac{dy}{dx} = \frac{e^2}{\sin x}\cos x + e^* \log \sin x \ = e^x \cos x + e^x \log \sin x$ $\frac{d^2y}{dx^2} = e^x \operatorname{cosec}^2 x + e^x \cos x + \frac{e^x}{\sin x}\cos x + e^x \log \sin x$ $\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-\operatorname{cosec}^2 x + \cot x)$
Ans. 30	(C) **Explanation :** Comer points will be (0,0), (2,7), (0,17/2) and (9,0) After putting these points in Z, we will get a maximum value of an objective function at (9,0) i.e, 540 .
Ans. 31	(A) Explanation: Given, $y = \frac{k\cos\theta - \sin\theta}{\sqrt{\cos^2\theta - \cos 2\theta}}$ We can simplify the given function, $y = \frac{k\cos\theta - \sin\theta}{\sqrt{\cos^2\theta - (\cos^2\theta - \sin^2\theta)}} = \frac{k\cos\theta - \sin\theta}{\sin\theta} = k\cot\theta - 1$ $\frac{dy}{d\theta} = -k\operatorname{cosec}^2\theta$ $\left.\frac{dy}{d\theta}\right]_{x=30^\circ} = -k \times 2 = 1$ $k = \frac{-1}{4}$

Ans. 32	(B) **Explanation:** We have, $f(x) = \dfrac{1}{x^2 - 2} = (x^2 - 2)^{-1}$ $f'(x) = -(x^2 - 2)^{-2} \cdot 2x = \frac{-2x}{(x^2-2)^2}$ for critical points, $f'(x) \ = 0$ $\dfrac{-2x}{(x^2 - 2)^2} = 0$ $x = 0$ when $x < 0, f'(x) >$ and when $x > 0, f'(x) < 0$
Ans. 33	(C) **Explanation:** $f: R \rightarrow R$ is defined as $f(x) = x^4$ Let $x, y \in R$ such that $f(x) = f(y)$ $\Rightarrow x^4 = y^4$ $\Rightarrow x = \pm y$ $\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$ For instance, $f(1) = f(-1) = 1$ $\therefore$ f is not one-one. Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that $f(x) = 2$. $\therefore$ f is not onto. Hence, function f is neither one-one nor onto. The correct answer is D.
Ans. 34	(B) **Explanation:** f is defined for all real values then, (i) $x + 11 = 0$ i.e., when $x = -11$ and (ii) $x^2 - 1 = 0$ i.e., when $x = -1, 1$ Hence, domain of $f = R - \{-11, -1, 1\}$

Ans. 35 (D)

Explanation:

Corner Point	Corresponding value of **Z**
(0,0)	0
(30,0)	120 Maximum
(20,30)	110
(0,50)	150

Hence, maximum value of Z is 120 at the point (30,0).

Ans. 36 (A)

Explanation:

$f: A \times B \rightarrow B \times A$ is defined as $f(a, b) = (b, a)$

Let $(a_1, b_1) \cdot (a_2, b_2) \in A \times B$

such that $f(a_1, b_1) = f(a_1, b_1)$

$\Rightarrow \ (b_1, a_1) = (b_2, a_2)$

$\Rightarrow \ b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow \ (a_1, b_1) = (a_2, b_2)$

$\therefore$ f is one-one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [By definition of f]

$\therefore$ f is onto.

Hence, function is Bijective.

Ans. 37 (B)

Explanation: $\frac{dy}{d\theta} = a(1 - \cos\theta)$

$\frac{dx}{d\theta} = a(0 - \sin\theta)$

	$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1-\cos\theta)}{-a\sin\theta} = \frac{\cos\theta - 1}{\sin\theta}$
Ans. 38	(C) Explanation: $\lim_{x\to 3/2} f(x) = f(3/2)$ $\Rightarrow \lim_{x\to 3/2} f(x) = k$ $\Rightarrow \lim_{x\to\frac{3}{2}} \frac{(2x-3)(2x+3)}{2x-3} = k$ $\Rightarrow k = 6$
Ans. 39	(D) **Explanation:** The minimum value of Z occurs at (0,8).
Ans. 40	(A) **Explanation:** $f(x) = 3x + x^3 - \frac{1}{8}\sin^2 x$ $f'(x) = 3 + 3x^2 - \frac{1}{8}2\sin x\cos x = 3 + 3x^2 - \frac{1}{8}\sin 2x$ We know the maximum value of $\sin x$ is 1 Even if we consider the maximum value of i.e., $\sin x = 1$ $f'(1) > 0$ $f'(0.5) > 0$ $f'(3) > 0$ $\therefore$ Function is increasing.

Corner Points	$Z = 25x_1 + 20x_2$
(8,0)	200
(52,154)	4,380
(0,8)	160 (Minimum)

	Section - C
Ans. 41	(B)
Ans. 42	(B) **Explanation :** Let us assume that, $f(x) = \sin x \cos x$ Now, we know that $\sin x \cdot \cos x = \frac{1}{2}\sin 2x$ $\therefore\ f'(x) = \frac{1}{2} \cdot \cos 2x.2 = \cos 2x$ Now, $f'(x) = 0$ $\Rightarrow\ \cos 2x = 0$ $\Rightarrow\ \cos 2x = \cos\frac{\pi}{2}$ $\Rightarrow\ x = \frac{\pi}{4}$ Also , $f''(x) = \frac{d}{dx} \times \cos 2x = -2 \times \sin 2x$ $\therefore [f''(x)]_{atx=\frac{\pi}{4}} = -2\sin 2 \times \frac{\pi}{4}$ $= -2\sin\frac{\pi}{2} = -2 < 0$ $\therefore x = \frac{\pi}{4}$ is point of maxima. $f\left(\frac{\pi}{4}\right) = \frac{1}{2} \times \sin \times 2 \times \frac{\pi}{4} = \frac{1}{2}$
Ans. 43	(C) **Explanation:** Let $y = \left(\frac{1}{x}\right)^x \Rightarrow\ \log y = x \cdot \log\frac{1}{x}$ $\therefore\ \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) + \log\frac{1}{x}.1 = -1 + \log\frac{1}{x}$

	$\therefore \ \frac{dy}{dx} = \left(\log\frac{1}{x} - 1\right) \cdot \left(\frac{1}{x}\right)^{x}$ $\Rightarrow$ Now , $\frac{dy}{dx} = 0$ $\Rightarrow \log\frac{1}{x} = 1 = \log e$ $\Rightarrow \ \frac{1}{x} = e$ $\Rightarrow \ x = \frac{1}{e}$ Hence, the maximum value of $\left(\frac{1}{x}\right)^{x} = (e)^{1/e}$.
Ans. 44	(C)
Ans. 45	(B) **Explanation:** It is given that $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ Equating the corresponding elements, we get $3x + 7 = 0 \Rightarrow \ x = -\frac{7}{3}$ $5 = y - 2 \Rightarrow \ y = 7$ $y + 1 = 8$ $\Rightarrow \ y = 7$ $2 - 3x = 4$ $\Rightarrow \ x = -\frac{2}{3}$ We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible. Hence, it is not possible to find the values of x and y for which the given matrices are equal.
Ans. 46	(A) **Explanation:** Expense of family A

	$= [3 \quad 2 \quad 1]\begin{bmatrix}200\\100\\50\end{bmatrix} = [850]$
Ans. 47	(C) **Explanation:** Expense of family C $= [4 \quad 3 \quad 2]\begin{bmatrix}200\\100\\50\end{bmatrix} = [1200]$
Ans. 48	(B) **Explanation:** Combined expense of women $= [2 \quad 4 \quad 3]\begin{bmatrix}100\\100\\100\end{bmatrix} = [900]$
Ans. 49	(C) **Explanation:** Most expensive family is C with an expense of Rs 1200.
Ans. 50	(B) **Explanation:** $[3 \quad 2]\begin{bmatrix}200\\50\end{bmatrix} = [700]$