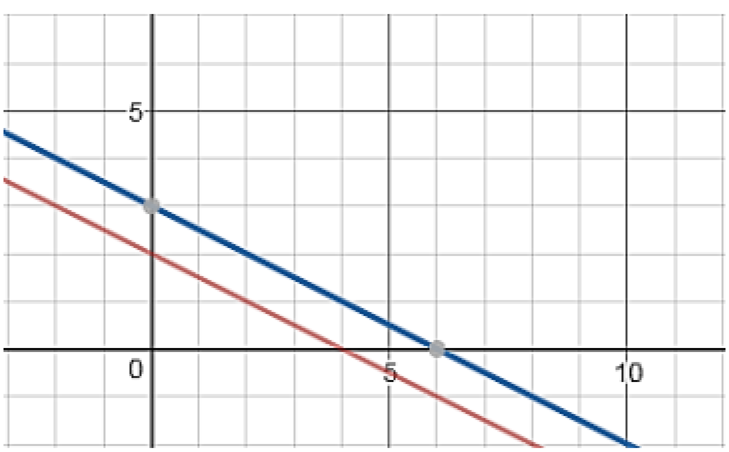
Board –CBSE | Class –10th | Topic – Linear Equation in two variables

(1) An equation in the form $ax + by + c = 0$, where a, b and c are real numbers, and $a \neq 0$ and $b \neq 0$, is called a linear equation in two variables x and y.
For Example: $2x + 3y + 7 = 0$, where $a = 2,\ b = 3, c = 5$ are real numbers. So, the given equation is a linear equation in two variables.

(2) Each solution $(x,\ y)$ of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.
For example: $2x + 3y = 5$ has $(1, 1)$ as its solution. So, $(1, 1)$ will lie on the line $2x + 3y = 5$

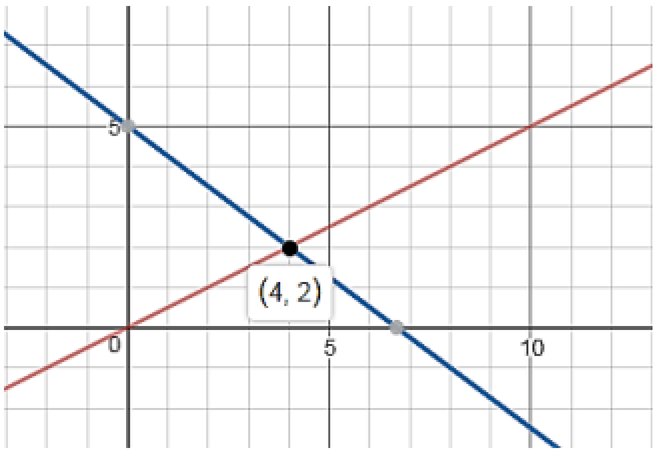
(3) The general form for a pair of linear equations in two variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1,\ b_1,\ c_1,\ a_2,\ b_2,\ c_2$ are all real numbers and $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$
For example: $2x + 3y - 7 = 0$ and $9x - 2y + 8 = 0$ form a pair of linear equations.



(4) A pair of linear equations that has no solution is called an inconsistent pair of linear equations. In this case, the lines may be parallel $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
For example: $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ are parallel lines.

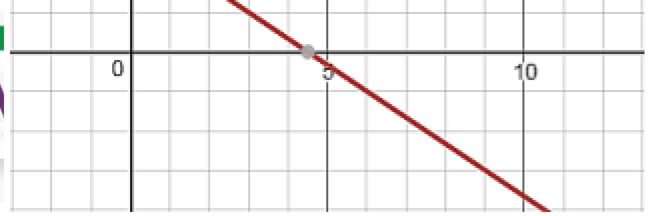
(5) A pair of linear equations in two variables, which has a solution, is called a consistent pair of linear equations. In this case, the lines may intersect in a single point and $a_1/a_2 \neq b_1/b_2$

For example: $x - 2y = 0$ and $3x + 4y - 20$ intersect each other at a unique point (4, 2)



(6) A pair of linear equations which are equivalent has infinitely many distinct common Solutions. Such a pair is called a dependent pair of linear equations in two variables. In this case, the lines may be coincident and $a_1/a_2 = b_1/b_2 = c_1/c_2$

For example: $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ are coincident lines.



(7) Algebraic Methods of solving a Pair of Linear Equations:

(i) Substitution Method

Follow the steps given below to understand the Substitution Method:

Step 1: Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

Step 2: Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved. Sometimes, one can get statements with no variable. If this statement is true, you can conclude that the pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.

Step 3: Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

For example: Solve the following pair of equations by substitution method: $7x - 15y = 2$ and $x + 2y = 3$

We can rewrite $x + 2y = 3$ as $x = 3 - 2y$ $\quad\quad - (1)$

Substituting the value of x in $7x - 15y = 2$, we get,

$7(3 - 2y) - 15y = 2$

$21 - 14y - 15y = 2$

$- 29\,y = \quad - 19$

Thus, $y = 19/29$

Now, substituting the value of ‘y’ in (1), we get, $x = 3 - 2(19/29) = 49/29$

(ii) Elimination Method:

Follow the steps given below to understand Elimination Method:

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2: Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3 . If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions. If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.

Step 3: Solve the equation in one variable (x or y) so obtained to get its value.

Step 4: Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

For example: Solve the following pair of equations by elimination method:

$9x - 4y = 2000$ and $7x - 3y = 2000$

Multiplying $9x - 4y = 2000$ by 3 and $7x - 3y = 2000$ by 4,

we get, $27x - 12y = 6000$ and $28x - 12y = 8000$

Subtracting both of these equations, we get,

$(28x - 27x) - (12y - 12y) = 8000 - 6000$

$x = 2000$

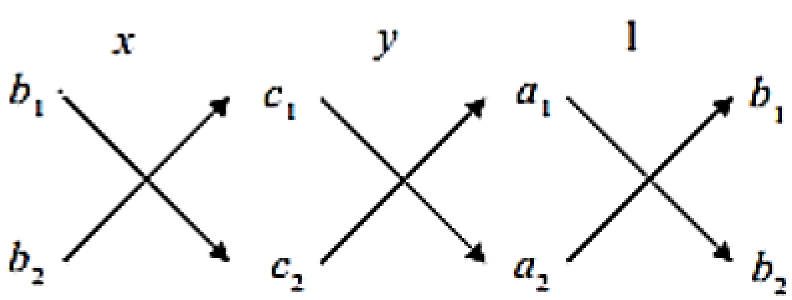
Substituting the value of x in $9x - 4y = 2000$, we get,

$9(2000) - 4y = 2000$

$y = 4000$

(iii) Cross Multiplication Method:

Follow the steps given below to understand Cross Multiplication Method:



Step 1: Write the given equations in the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Step 2: Take the help of the diagram below And write the equations as shown below

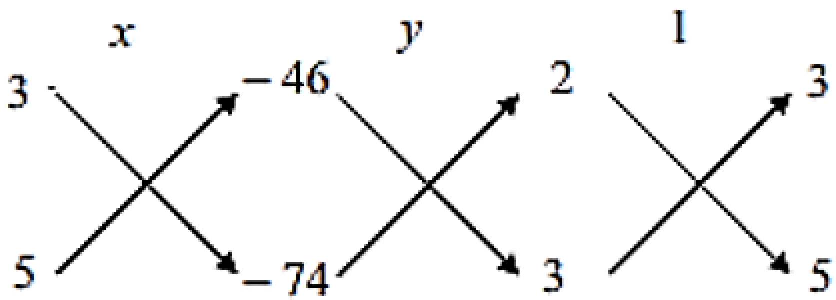
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Step 3: Find x and y, provided $a_1b_1 - a_2b_1 \neq 0$

For example: Solve the following pair of equations by cross multiplication method:

$2x + 3y - 46 = 0$ and $3x + 5y - 74 = 0$

Given equations are in the form of $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$



Now, with the help of the diagram, we can rewrite equations as

$x/((3)(-74) - (5)(-46) = y/((-46)(3) - (-74)(2) = 1/((2)(5) - (3)(3)$

$$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{\frac{1}{(10-9)x}}{8} = \frac{y}{10} = \frac{1}{1}$$

$Thus\ x = 8\ and\ y = 10$

(8) Equations Reducible to a Pair of Linear Equations in Two Variables:

Let us understand it by an example:

For example: Solve $(2/x) + (3/y) = 13$ and $(5/x) - (4/y) = -2$

We can rewrite the given equations as, $2(1/x) + 3(1/y) = 13$ and $5(1/x) - 4(1/y) = -2$

Let us substitute $1/x = p$ and $1/y = q$

so we get, $2p + 3q = 13$ and $5p - 4q = -2$

On solving these equations, we get, $p = 2$ and $q = 3$.

We know, $p = 1/x = 2$ and $q = 1/y = 3$

Thus, $x = 1/2$ and $y = 1/3$.