Board –CBSE | Class –10th | Topic – Area Related To Circle

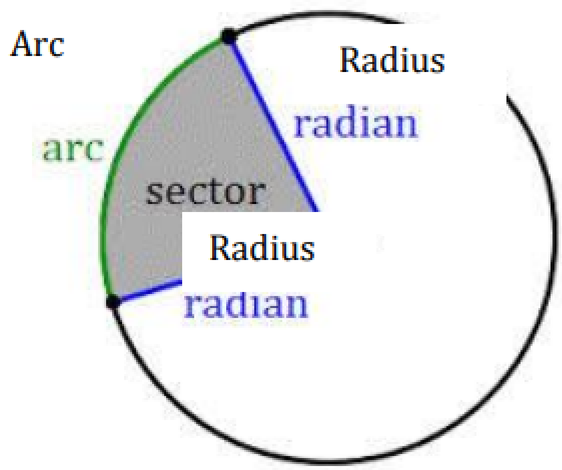
- For a circle of a radius r, we have

  (i) Circumference $= 2\pi r$

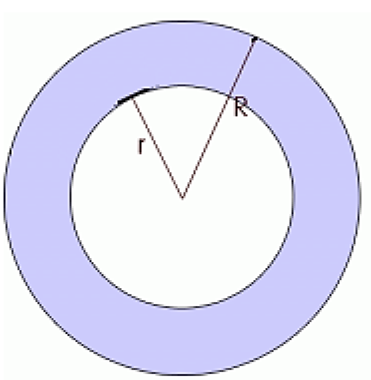
  (ii) Area $= \pi r^2$

  (iii) Area of semi-circle $= \frac{\pi r^2}{2}$

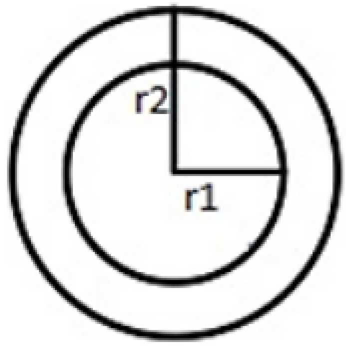
  (iv) Area of a quadrant $= \frac{\pi r^2}{4}$



- If R and r are the radii of two concentric circles such that $R > r$ then, then the area enclosed by the two circles $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$



- For example: The area enclosed between the concentric circle is 770 cm^2. If the radius of the the outer circle is 21 cm, find the radius of the inner circle.



  Solution: Let the radius of the inner and outer radius be r_1 and r_2 respectively.

  It is given that the area enclosed between concentric circles is 770 cm^2

  The radius of the outer circle is 21 cm

  Then, the area enclosed between the concentric circle = $\pi r_2^2 - \pi r_1^2$

  $\Rightarrow \pi r_2^2 - \pi r_1^2 = 770$

  $\Rightarrow \pi\left((21) - r_1^2\right) = 770$

  $\Rightarrow \left(441 - r_1^2\right) = \frac{770 \times 7}{22}$

  $\Rightarrow r_1^2 = 441 - 245 = 196$

  $\Rightarrow r_1 = 14$

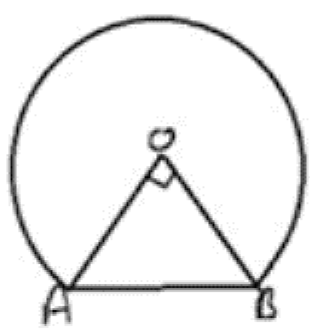
  Hence, the radius of the inner circle is 14 cm.

- If a sector of a circle of radius r contains an angle of θ Then,

  (i) Length of the arc of the sector = $\frac{\theta}{360^\circ} \times 2\pi r = \frac{\theta}{360^\circ} \times$ Circumference

  (ii) Perimeter of the sector = $2r + \frac{\theta}{360^\circ} \times 2\pi r$

  Example: The radius of the circular part of the cross-section of a railway tunnel is 2 m. If $\angle AOB = 90^\circ$ calculate the height and perimeter of the cross-section.



  Solution:

We have $OA = 2\,m$

Now using Pythagoras theorem in $\Delta AOB,\ AB = \sqrt{2^2 + 2^2} = 2\sqrt{2}\,m$

Let the height of the tunnel be h

Area of $\Delta AOB = \frac{1}{2} \times 2 \times 2 = \frac{1}{2} \times 2\sqrt{2} \times 0M = 2$

$OM = \sqrt{2}$

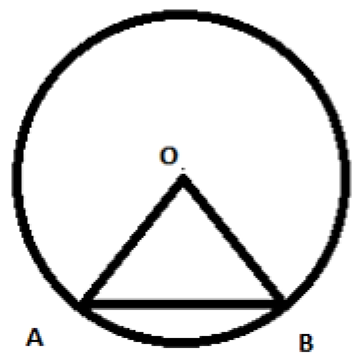
$h = \left(2 + \sqrt{2}\right)cm$

$The\ perimeter\ of\ cross - section = major\ arc\ AB + AB = \left(2\pi \times 2 \times \frac{3}{4}\right) + 2\sqrt{2}$

$$= (3\pi + 2\sqrt{2})\ cm$$

(iii) Area of the sector $= \frac{\theta}{360^{\circ}} \times \pi r^2 = \frac{\theta}{360^{\circ}} \times$ (The area of the circle)

Example: AB is a chord of a circle with center O and a radius of 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment



Solution:

It is given that chord AB divides the circle into two segments In ΔAOB,

$OA = OB = 4\ cm$

$AM = \frac{AB}{2} = 2\ cm$

Let $\angle AOB = 2\theta$, then

$\angle AOM = \angle BOM = \theta$

In $\angle OAM$ we have

$sin\,\theta = \frac{AM}{A0} = \frac{2}{4} = \frac{1}{2}$

$\theta = \frac{1}{2}$

$= 30°$

Hence, $\angle AOB = 2\theta = 2\times30^{\circ} = 60^{\circ}$

We know that the area of a minor segment of angle θ in a circle of radius r is

$A = \left\{\frac{\pi\theta}{360} - sin\,\frac{\theta}{2}\,cos\,\frac{\theta}{2}\right\}r^{2}$

Now, using the value of r and θ we can find the area of the minor segment:

$A = \left\{\frac{\pi\theta}{360} - \sin sin\,\frac{60^{\circ}}{2}\cos cos\,\frac{60^{\circ}}{2}\right\}(4)^{2}$

$\Rightarrow A = \left\{\frac{\pi}{6} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right\}(4)^{2}$

$A = \left\{\frac{8\pi}{3} - 43 - \sqrt{3}\right\}cm^{2}$

Hence, the area of the minor segment is $\left\{\frac{8\pi}{3} - 4 - \sqrt{3}\right\}cm^{2}$

(iv)Area of the segment = Area of the corresponding sector - Area of the corresponding triangle

$$= \frac{\theta}{360}\times\pi r^{2} - r^{2}sin\,\frac{\theta}{2}\,cos\,\frac{\theta}{2} = \left\{\frac{\pi\theta}{360} - sin\,\frac{\theta}{2}cos\,\frac{\theta}{2}\right\}r^{2}$$

Example: The radius of a circle with center O is 5 cm. two radii OA and OB are drawn at right angles to each other. Find the area of the segment made by chord AB.

Solution: Radius of the circle = 5 cm

Area of the minor segment, $AB = \left(\frac{\pi\theta}{360^{\circ}} - sin\,\frac{\theta}{2}cos\,\frac{\theta}{2}\right)(r)^{2}$

$AB = \left(\frac{3.14\times90^{\circ}}{360^{\circ}} - \sin 45^{\circ}\cos 45^{\circ}\right)(5)^{2}$

$= \left(\frac{282.6}{360^{\circ}} - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)(5)^{2}$

$= 7.125\ cm^{2}$

$Area\ of\ the\ minor\ segment\ = area\ of\ the\ circle\ - area\ of\ the\ minor\ segment\ = \pi r^{2} - 7.125$

$= 3.14\times25 - 7.125$

$= 71.37\ cm^{2}$