Board – CBSE | Class – 10th | Topic – Arthimatic Progression

1. A sequence is an arrangement of numbers or objects in a definite order.

Example: 1, 8, 27, 64, 125….

In the above arrangement, numbers are arranged in a definite order according to some rules.

2. If 'a' is the first term and 'd' the common difference of an AP, then the A.P. is

$a,\ a + d,\ a + 2d,\ a + 3d,\ a + 4d$…

Example: If the AP is 2, 4, 6, 8, ….

Then first term $a = 2$ and $d = 2$

So, 2, $2 + 4,\ 2 + 2(2),\ 2 + 3(2),\ 2 + 4(2)$…

3. A sequence $a_1, a_2, a_3, \ldots, a_n$, is an AP if $a_{n+1} - a_n$ is independent of n.

Example: If the sequence is 2, 4, 6, 8, ……a_n, ….

if we take $a_n = 16$ so $a_{n+1} = 18$ so a_{n+1} - $a_n = 18 - 16 = 2$ which is independent of n

4. A sequence $a_1, a_2, a_3, \ldots, a_n$. is an AP, if and only if its n^{th} term a_n is a linear expression in n and a. In such a case, the coefficient of n is the common difference.

Example: A sequence 1, 4, 9, 16, 25, … is an AP. Suppose n^{th} term is $a_n = 81$ that is a linear expression in n, which is n^2.

5. The n^{th} term a_n, of an AP with the first term 'a' and common difference 'd' is given by $a_n = a + (n - 1)d$

Example: If want to find the n^{th} term a_n in the example given in 4^{th} point, we know that $a = 2,\ and\ d = 2$ then we can find the 10^{th} term by putting $n = 10$ in the above equation. So the 10^{th} term of the sequence is $a_{10} = 2 + (10 - 1)2 = 20$

6. Let there be an A.P with the first term 'a' and common difference 'd'. If there are m terms in the AP, then,

n^{th} term from the end $= (m - n + 1)^{th}$ term from the beginning $= a + (m - n)d$

Also, the n^{th} term from the end = Last term $+ (n - 1)(-d)$

$= l - (n - 1)d$, where ' l ' denotes the last term.

Example: Determine the 10^{th} term from the end of the A.P 4, 9, 14, …, 254.

Here, it is given that $l = 254,\ d = 5$

n^{th} term from the end $= l - (10 - 1)d = l - 9d = 254 - 9\times5 = 209$

7. Various terms in an AP can be chosen in the following manner.

No. of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a +2d	d
6	a – 5d, a – 3d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

8. The sum to n terms of an A.P with first term 'a' and common difference 'd' is given by $S_n = \frac{n}{2}\{2a + (n - l)d\}$

Also, $S_n = \frac{n}{2}\{a + l\}$, where l is last term $= a + (n - l)d$

9. If the ratio of the sums of n terms of two AP's is given, then to find the ratio of their n^{th} terms, we replace n by $(2n - 1)$ in the ratio of the sums of n terms.

Example: The ratio of the sum of n terms of two AP's is $(7n + 1): (4n + 27)$

Find the ratio of their m^{th} terms.

Solution:

let a_1, a_2 be the 1^{st} terms and d_1, d_2 the common differences of the two given A.P's, then the sums of their n terms are given by,

$$S_{n_1} = \frac{n}{2}\{2a_1 + (n - 1)d_1\} \text{ and } S_{n_2} = \frac{n}{2}\{2a_2 + (n - 1)d_2\}$$

$$\frac{S_{n1}}{S_{n2}} = \frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}}$$

$$\frac{S_{n1}}{S_{n2}} = \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}$$

It is given that $\frac{S_{n1}}{S_{n2}} = \frac{7n+1}{4n+27}$

$$\frac{7n + 1}{4n + 27} = \frac{2a_1+(n - 1)\, d_1}{2a_2+(n - 1)\, d_2} \text{……………. } (i)$$

To find the ratio of the m^{th} terms of the two given AP's, we replace n by $(2m - 1)$ in equation (i). Therefore,

$$\frac{7(2m-1) + 1}{4(2m-1) + 27} = \frac{2a_1 + ((2m-1) - 1)\, d_1}{2a_2+((2m-1) - 1)\, d_2}$$

$$\frac{14m-6}{8m+23} = \frac{a_1 + (m-1)\, d_1}{a_2 + (m-1)\, d_2}$$

Hence, the ratio of the m^{th} terms of the two AP's is $(14m - 6): (8m + 23)$

10. A sequence is an AP if and only if the sum of its n terms is of the form $An^2 + Bn$, where A, B are constants. In such a case the common difference is $2A$.

Example: (i)
For the A.P

$$S_n = pn + q^2 n$$

Now $S_1 = p\times1 + q(1)^2$

$S_1 = p + q \Rightarrow T_1 = p + q$ and also $S_2 = p\times2 + q(2)^2$

$S_2 = 2p + 4q$

We have $T_1 + T_2 = 2p + 4q$

Or $T_2 = 2p + 4q - T_1$

$T_2 = 2p + 4q - (p + q) \Rightarrow p + 3q$

Hence common difference $= T_2 - T_1$

$= p + 3q - (p + q) = 2q$

Example: (ii) 50, 46, 42, ... find the sum of the first 10 terms

Solution:

Given, 50, 46, 42, ...

Here, first term $a = 50$

Difference $d = 46 - 50 = (-4)$

And no of terms $n = 10$

We know $S_n = n/2[2a + (n - 1)d]$

Sn $= 10/2[2(50) + (10 - 1)(-4)] \Rightarrow 5[100 + (9)(-4)]$

$S_n = 5[100 - 36] \Rightarrow 5 \times 64 \Rightarrow 320$

Hence, the sum of the first 10 terms is 320.

(iii) The first term is 17 and the last term is 350 and $d = 9$. Find the total sum and the number of terms.

Solution:

Given, first term, $a = 17$, last term, $a_n = 350 = l$

And difference $d = 9$

We know, $a_n = a + (n - 1)d$

$350 = 17 + (n - 1)9$

$350 = 17 + 9n - 9$

$9n = 350 - 17 + 9 \Rightarrow 342$

$n = 38$

We know, the sum of n terms,

$S_n = n_2(a + l)$

$S_{38} = 38/2[17 + 350] \Rightarrow 19 \times 367 \Rightarrow 6973$

Hence, the number of terms is 38 and the sum is 6973.