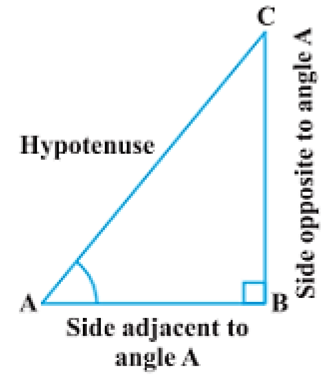
Board –CBSE	Class –10th	Topic – Introduction to Trigonometry

- For a right angled triangle ABC, side BC is called the side opposite to angle A, AC is called the hypotenuse and AB is called the side adjacent to angle A.
- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides. The trigonometric ratios of angle A in right triangle ABC are defined as follows:



- sine of $\angle A = \frac{side\ opposite\ to\ angle\ A}{hypotenuse} = \frac{BC}{AC}$
- cosine of $\angle A = \frac{side\ adjacent\ to\ angle\ A}{hypotenuse} = \frac{AB}{AC}$
- tangent of $\angle A = \frac{side\ opposite\ to\ angle\ A}{side\ adjacent\ to\ angle\ A} = \frac{BC}{AB}$
- cosecant of $\angle A = \frac{1}{sine\ of\ \angle A} = \frac{hypotenuse}{side\ opposite\ to\ angle\ A} = \frac{AC}{BC}$
- secant of $\angle A = \frac{1}{cosine\ of\ \angle A} = \frac{hypotenuse}{side\ adjacent\ to\ angle\ A} = \frac{AC}{AB}$
- cotangent of $\angle A = \frac{1}{tangent\ of\ \angle A} = \frac{side\ adjacent\ to\ angle\ A}{side\ adjacent\ to\ angle\ A} = \frac{AB}{AC}$

- Trigonometric Ratios of 45°:

  In $\triangle ABC$, right-angled at B, if one angle is 45°, then the other angle is also 45°, i.e., $\angle A = \angle C = 45^\circ$.

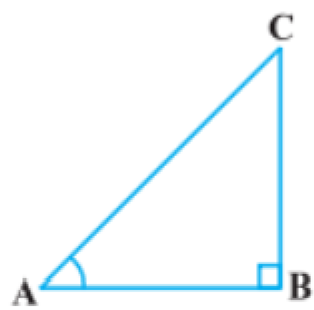
  Now, let $BC = AB = a$.

  Applying the Pythagoras Theorem, we get,

  $AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

  So, $AC = a\sqrt{2}$

  As per the trigonometric ratio definitions, we have,



- $sin\ 45^{\circ} = \frac{side\ opposite\ to\ angle\ 45^{\circ}}{hypotenuse} = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$
- $cos\ 45^{\circ} = \frac{side\ adjacent\ to\ angle\ 45^{\circ}}{hypotenuse} = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$
- $tan\ 45^{\circ} = \frac{side\ opposite\ to\ angle\ 45^{\circ}}{side\ adjacent\ to\ angle\ 45^{\circ}} = \frac{BC}{AB} = \frac{a}{a} = 1$
- $cosec\ 45^{\circ} = \frac{1}{sin\ 45^{\circ}} = \frac{hypotenuse}{side\ opposite\ to\ angle\ 45} = \frac{AC}{BC} = \sqrt{2}$
- $sec\ 45^{\circ} = \frac{1}{cos\ 45^{\circ}} = \frac{hypotenuse}{side\ adjacent\ to\ angle\ 45^{\circ}} = \frac{AC}{AB} = \sqrt{2}$
- $cot\ 45^{\circ} = \frac{1}{tan\ 45^{\circ}} = \frac{side\ adjacent\ to\ angle\ 45}{side\ opposite\ to\ angle\ 45^{\circ}} = \frac{AB}{BC} = 1$

- Trigonometric Ratios of 30° and 60° :

  Consider an equilateral triangle ABC. Since each angle in an equilateral triangle is 60°, therefore, $\angle A = \angle B = \angle C = 60^{\circ}$

  And draw a perpendicular AD from A to side BC.

  Here, $\Delta ABD \cong \Delta ACD$. Therefore, $BD = DC$ and $\angle BAD = \angle CAD$

  (as per CPCT)

  From the figure, it can be seen that, $\angle ABD = 60^{\circ}$ and

  $\angle BAD = \frac{1}{2}\angle BAC = \frac{1}{2} \times 60^{\circ} = 30^{\circ}$

Now, let us assume $AB = BC = CA = 2a$. Therefore, $BD = \frac{1}{2} \times BC = a$. Applying Pythagoras theorem in ΔABD, we get,

$AD^{2} = AB^{2} - BD^{2} = (2a)^{2} - (a)^{2} = 3a^{2}$

Hence, $AD = \sqrt{3}$ a

As per the trigonometric ratio definitions, we have,

- $sin\ 30^{\circ} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

- $cos\ 30^{\circ} = \frac{AD}{AB} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$
- $tan\ 30^{\circ} = \frac{BD}{AD} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$
- $cosec\ 30^{\circ} = \frac{1}{sin\ 30^{\circ}} = 2$
- $sec\ 30^{\circ} = \frac{1}{cos\ 30} = \frac{2}{\sqrt{3}}$
- $cot\ 30^{\circ} = \frac{1}{tan\ 30^{\circ}} = \sqrt{3}$

- Similarly, as per the trigonometric ratio definitions, we have,

  (i) $sin\ 60^{\circ} = \frac{\sqrt{3}}{2}$

  (ii) $cos\ 60^{\circ} = \frac{2}{1}$

  (iii) $tan\ 60^{\circ} = \sqrt{3}$

  (iv) $cosec\ 60^{\circ} = \frac{2}{\sqrt{3}}$

  (v) $sec\ 60^{\circ} = 2$

  (vi) $cot\ 60^{\circ} = \frac{1}{\sqrt{3}}$

- Trigonometric Ratios of 0° :

  (i) $Sin\ 0^{\circ} = 0$

  (ii) $Cos\ 0^{\circ} = 1$

  (iii) $Tan\ 0^{\circ} = 0$

  (iv $Cosec\ 0^{\circ}$ = Not defined

  (v) $Sec\ 0^{\circ} = 1$

  (vi) $Cot\ \ 0^{\circ}$ = Not defined

- Trigonometric Ratios of 90° :

  (i) $sin\ 90^{\circ} = 1$

  (ii) $cos\ 90^{\circ} = 0$

  (iii) $tan\ 90^{\circ}$ = Not defined

  (iv) $cosec\ 90^{\circ} = 1$

  (v) $sec\ 90^{\circ}$ = Not defined

  (vi) $cot\ 90^{\circ} = 0$

- Table representing the trigonometric ratios of $0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$, and 90°:

∠A	0°	30°	45°	60°	90°
sin A	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1/2	0
tan A	0	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$		2	Not defined
cot a	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example: Evaluate $2tan^2 45^\circ + cos^2 30^\circ - sin^2 60^\circ$

Now, $2tan^2 45^\circ + cos^2 30^\circ - sin^2 60^\circ$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4}$

$= 2$

Trigonometric Ratios of Complementary Angles:

- $sin\left(90^\circ - A\right) = cosA$
- $cos\left(90^\circ - A\right) = sinA$
- $tan\left(90^\circ - A\right) = cotA$
- $cot\left(90^\circ - A\right) = tanA$
- $sec\left(90^\circ - A\right) = cosecA$
- $Cosec\left(90^\circ - A\right) = secA$

Example: Simplify: $-\ \frac{tan\ 26^\circ}{cot\ 64^\circ}$

We know that, $cot\ A\ = tan\left(90^\circ - 64^\circ\right) = cot\ 64^\circ$.

Therefore, $\frac{tan\ 26^\circ}{cot\ 64^\circ} = \frac{cot\ 64^\circ}{cot\ 64^\circ} = 1.$

Trigonometric Identities:

- $cos^2 A + sin^2 A = 1$
- $1 + tan^2 A = sec^2 A$
- $cot^2 A + 1 = cosec^2 A$

Example: Evaluate $sin\,25^{\circ} cos\,65^{\circ} + cos\,25^{\circ} sin\,65^{\circ}$

$= sin\,25^{\circ} cos\,65^{\circ} + cos\,25^{\circ} sin\,65^{\circ}$

$= sin\,25^{\circ} cos\left(90^{\circ} - 25^{\circ}\right) + cos\,25^{\circ} sin\left(90^{\circ} - 25^{\circ}\right)$

$= sin^{2} 25^{\circ} + cos^{2} 25^{\circ}$

$= 1$ $\left(Since\ cos^{2}A + sin^{2}A = 1\right)$

Example: Prove that $(cosecA - sinA)(secA - cosA) = 1/(tanA + cotA)$

$LHS = (cosecA - sinA)(secA - cosA)$

$= \left(\frac{1}{sin\,A} - sin\,A\right)\left(\frac{1}{cos\,A} - cos\,A\right)$

$= \frac{1-sin^{2}A}{sinA} \times \frac{1-cos^{2}A}{cos\,A}$

$= \frac{cos^{2}A}{sin\,A} \times \frac{sin^{2}A}{cos\,A}$ (Since, $1 - cos^{2}A = sin^{2}A$, and $1 - sin^{2}A = cos^{2}A$)

$= cos\,A\,sin\,A = sin\,A\,cos\,A$

$RHS = \frac{1}{tan\,A + cot\,A}$

$= \frac{1}{\frac{sin\,A}{cos\,A} + \frac{cos\,A}{sin\,A}}$

$= \frac{1}{\frac{sin^{2}A + cos^{2}A}{sin\,A\,cos\,A}}$ (Since, $sin^{2}A + cos^{2}A = 1$)

$= sin\,A\,cos\,A$

Thus, LHS = RHS.