Board –CBSE | Class –10th | Topic – Quadratic Equations

(1) A polynomial of degree 2 is called a quadratic polynomial. The general form of a quadratic polynomial is $ax^2 + bx + c$, where a, b, c are real numbers such that $a \neq 0$ and x is a real variable.

Example: $x^2 + 5x + 3$, where $a = 1, b = 5, c = 3$ are real numbers. So the given equation is a quadratic polynomial.

(2) If $p(x) = ax^2 + bx + c,\ a \neq 0$ is a quadratic polynomial and α is a real number, then $p(\alpha) = a\alpha^2 + b\alpha + c$ is known as the value of the quadratic polynomial $p(\alpha)$.

Example: $p(\alpha) = \alpha^2 + 5\alpha + 3$, in the given equation if $\alpha = 3$ then $p(\alpha) = 27$. So 27 is a value of quadratic polynomial

(3) A real number α is said to be a zero of quadratic polynomial $p(x) = ax^2 + bx + c$ if $p(\alpha) = 0$.

Example: $p(x) = x^2 + 6x + 5$

If $x = (-5)$ then $p(x) = 0$, So -5 is the zero of the polynomial.

(4) If $p(x) = ax^2 + bx + c$ is a quadratic polynomial, then $p(x) = 0$ i.e., $ax^2 + bx + c = 0,\ a \neq 0$ is called a quadratic equation.

Example: $p(x) = x^2 - 8x + 16,\ a = 1$, is called a quadratic equation.

(5) A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$.

In other words, α is a root of $ax^2 + bx + c = 0$ if and only if α is a zero of the polynomial $p(x) = ax^2 + bx + c$

Example: Suppose the quadratic equation is $2x^2 - x - 6 = 0$.

If we put $x = 2$ then $p(x) = 0$. So 2 is the root of that given equation and here $\alpha = 2$.

(6) If $ax^2 + bx + c = 0,\ a \neq 0$ is factorizable into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

Example: The given equation is $9x^2 - 3x - 2 = 0$

Now, solving the above equation using the factorization method,

$\Rightarrow 9x^2 - 6x + 3x - 2$

$\Rightarrow 3x(3x - 2) + 1(3x - 2)$

$\Rightarrow (3x + 1) = 0\ or\ (3x - 2) = 0$

$\Rightarrow 3x = -1\ or\ (3x - 2) = 0$

$\Rightarrow x = -\frac{1}{3}\ or\ x = \frac{2}{3}$

Hence $x = -\frac{1}{3}$ and $x = \frac{2}{3}$ are the two roots of the given equation.

(7) The roots of a quadratic equation can also be found by using the method of completing the square.

Example: The given equation is $-2x^2 - 7x + 3 = 0$

Dividing throughout by 2,

$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

Shifting the constant term to the right-hand side,

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

Adding square of the half of coefficient of x on both sides,

$\Rightarrow x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2$

$\Rightarrow x^2 - 2\frac{7}{4}x + \left(\frac{7}{4}\right)^2 =- \frac{3}{2} + \left(\frac{49}{16}\right)$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{-24+49}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

Taking the square root of both sides,

$\Rightarrow \left(x - \frac{7}{4}\right) = \sqrt{\frac{25}{16}}$

$\Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{5}{4} + \frac{7}{4}\ or\ x =- \frac{5}{4} + \frac{7}{4}$

$\Rightarrow x = 3\ or\ x = \frac{1}{2}$

Hence $x = 3$, and $x = 1/2$ are the two roots of the given equation.

(8) The roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ can be found by using the quadratic formula,

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

provided that $\sqrt{b^2 - 4ac} \geq 0$

Example: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$is the given equation in the form of $ax^2 - bx + c = 0$,

where $a = \sqrt{3}, b = 10c = 8\sqrt{3}$

Therefore, the discriminant: $D = b^2 - 4ac$

$D = (10)^2 - 4\times\sqrt{3}\times(- 8\sqrt{3})$

$D = 100 + 96$

$D = 196$

Since $D > 0$, the roots of the given equation are real and distinct.

The real roots α and β are given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-10+\sqrt{196}}{2\sqrt{3}} = 2\sqrt{\frac{3}{2}24} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\beta = \frac{-b-\sqrt{D}}{\beta^2 a - \frac{-1\sigma}{2\sqrt{3}} - 10 - \sqrt{196}} = \frac{-4x\sqrt{3}x\sqrt{3}}{\sqrt{3}} = \frac{-12}{\sqrt{3}} = -4\sqrt{3}$$

Hence α and β are the two roots of the given equation.

$\alpha = \frac{2}{\sqrt{3}}$ $\beta =- 4\sqrt{3}$

(9) Nature of the roots of quadratic equation

In $ax^2 + bx + c = 0,\ a \neq 0$ depends upon the value of $D = b^2 - 4ac$, which is known as the discriminant of the quadratic equation.

(10) The quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ has:

(i) Two distinct real different roots, if $D = b^2 - 4ac > 0$

Example: $16x^2 = 24x + 1$

$16x^2 - 24x - 1 = 0$

The given equation is of the form of $ax^2 + bx + c = 0$, where $a = 16, b =- 24, c =- 1$

Therefore, the discriminant: $D = b^2 - 4ac$

$D = (-24)^2 - 4\times16\times(-1)$

$D = 576 + 64$

$D = 640$

Since, $D > 0$

Therefore, the roots of the given equation are real and distinct.

The real roots α and β are given by,

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-24)+\sqrt{640}}{2x16} = \frac{24+\sqrt{64x10}}{32} = \frac{24+8\sqrt{10}}{32} = 8\left(\frac{3+\sqrt{10}}{32}\right) = \left(\frac{3+\sqrt{10}}{4}\right)$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-24)-\sqrt{640}}{2x16} = \frac{24-\sqrt{64x10}}{32} = \frac{24-8\sqrt{10}}{32} = 8\left(\frac{3-\sqrt{10}}{32}\right) = \left(\frac{3-\sqrt{10}}{4}\right)$$

Hence $\alpha = \left(\frac{3+\sqrt{10}}{4}\right)$ and $\beta = \left(\frac{3-\sqrt{10}}{4}\right)$ are the two roots of the given equation.

(ii) Two equal roots i.e. coincident real roots, if $D = 0; b^2 - 4ac = 0.$

Example: $2x^2 - 2\sqrt{6}x + 3 = 0$

The given equation is of the form of $ax^2 + bx + c = 0$, where $a = 2,\ b =- 2\sqrt{6},\ c = 3$

Therefore, the discriminant: $D = b^2 - 4ac$

$= \left(- 2\sqrt{6}\right)2 - 4\times2\times3$

$= 24 - 24 = 0$

Since $D = 0$,the roots of the given equation are real. The real and equal roots are given by

$- \frac{b}{2a}$

$\frac{-b}{2a} = \frac{-(-2\sqrt{6})}{2x2} = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$

$\frac{\sqrt{3}\sqrt{3}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

(iii) No real roots, if $D < 0;\ b^2 - 4ac < 0$

Example: The given equation is $x^2 + x + 2 = 0$

The given equation is of the form of $ax^2 + bx + c = 0$, where $a = 1,\ b = 1,\ c = 2$

Therefore, the discriminant:

$D = b^2 - 4ac$

$D = (1)^2 - 4\times1\times2$

$D = 1 - 8$

$D =- 7$

Since $D < 0$,the given equation has no real roots.