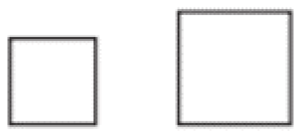
Board –CBSE	Class –10th	Topic – Triangles

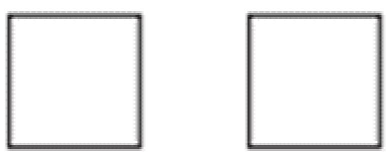
- Similar Figures: Two figures having the same shapes (and not necessarily the same size) are called similar figures.

  Example: The given below squares are similar.

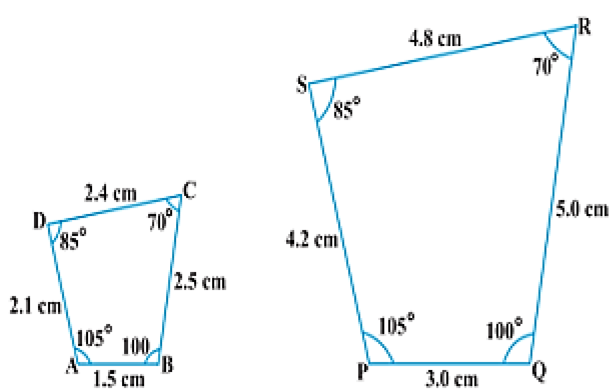


- Congruent Figures: The word 'congruent' means equal in all aspects of the figures whose shapes and sizes are the same.

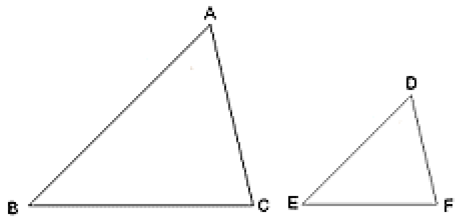
  Example: The given below squares are congruent.



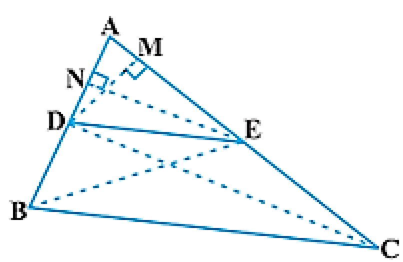
- Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion). Example: The given below quadrilaterals ABCD and PQRS are similar.



- Two triangles are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion). Example: The given below triangles ABC and DEF are similar.



- Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.
To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof: First, join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$.

Now, area of $\Delta ADE = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times AD \times EN$

So, $ar(ADE) = \frac{1}{2} \times AD \times EN$

$ar(BDE) = \frac{1}{2} \times DB \times EN$

$ar(ADE) = \frac{1}{2} \times AE \times DM$

$ar(DEC) = \frac{1}{2} \times EC \times DM$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ .... (1)

$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$

Here, $ar(BDE) = ar(DEC)$, since ΔBDE and DEC are on the same base DE and between the same parallels BC and DE

From (1),(2), and (3), we get, $\frac{AD}{DB} = \frac{AE}{EC}$

- Theorem: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

  Example: If a line intersects sides AB and AC of $\triangle ABC$ at D and E respectively and is parallel to BC,then prove that $\frac{AD}{AB} = \frac{AE}{AC}$

  To Prove: $\frac{AD}{AB} = \frac{AE}{AC}$

  Proof: Given, DE ∥ BC.

  Now, $\frac{AD}{AB} = \frac{AE}{AC}$ (Theorem 6.1) or, $\frac{DB}{AD} = \frac{EC}{AE}$

  Adding 1 on both the sides, we get,

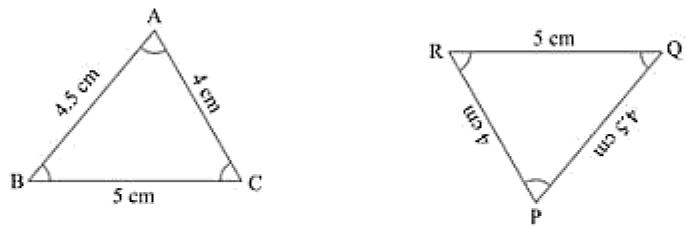
  $\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

  $\frac{AB}{AD} = \frac{AC}{AE}$

  Therefore, $\frac{AD}{AB} = \frac{AE}{AC}$

- Criteria for Similarity of Triangles: For triangles, if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle then they are said to be congruent triangles. The symbol '∼ ' stands for 'is similar to' and the symbol '≅' stands for 'is congruent to'.

Example: Consider two ΔABC and ΔPQR as shown below. Here, $\triangle ABC$ is congruent to Δ PQR which is denoted as $\triangle ABC \cong \triangle PQR$.



$\triangle ABC \cong \triangle PQR$ means sides $AB = PQ,\ BC = QR,\ CA = RP$; then $\angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R$ and vertices A corresponds to P, B corresponds to Q and C corresponds to R.

- Theorem: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar. This criterion is referred to as the AAA (Angle-Angle-Angle) criterion of similarity of two triangles.
  Given: Two triangles ABC and DEF exist such that $\angle A = \angle D,\ \angle B = \angle E$, and $\angle C = \angle F$.
  To Prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
  Proof: For the triangle DEF, we mark point P such that DP = AB and mark point Q such that $DQ = AC$. And join PQ.
  Hence, $\triangle ABC \cong \triangle DPQ$.
  Therefore, $\angle B = \angle P = \angle E$ and $PQ \parallel EF$.
  As per the theorem, we can say that $\frac{DP}{PE} = \frac{DQ}{QF}$ i. e. $\frac{AB}{DE} = \frac{BC}{EF}$
  Also, $\frac{AB}{DE} = \frac{BC}{EF}$
  Hence, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

- Theorem: If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. This criterion is referred to as the SSS (Side-Side-Side) similarity criterion for two triangles.

Example:

Given: Two triangles ABC and DEF exists such that

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} < 1$

To Prove: $\angle A = \angle D, \angle B = \angle E$, and $\angle C = \angle F$.

Proof: For the triangle DEF, we mark point P such that DP = AB and mark point Q such that $DQ = AC$. And join PQ

As per the theorem, we can say that, $DP/PE = DQ/QF$ and $PQ \parallel EF$.

Hence, $\angle P = \angle E$ and $\angle Q = \angle F$

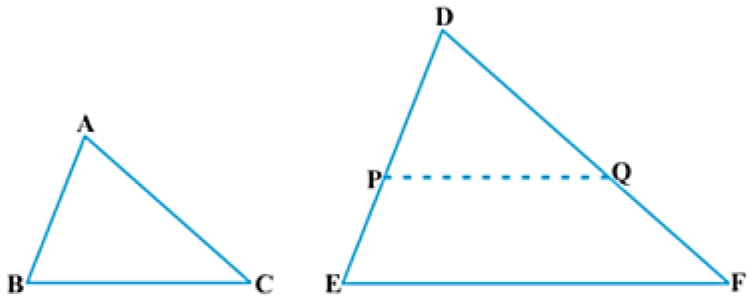
Also, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

Hence, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$ (As per CPCT)

On comparison, we get, $BC = PQ$.

Thus, $\Delta ABC \cong \Delta DPQ$.

Hence, So, $\angle A = \angle D, \angle B = \angle E$, and $\angle C = \angle F$



- Theorem: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This criterion is referred to as the SAS (Side-Angle-Side) similarity criterion for two triangles.

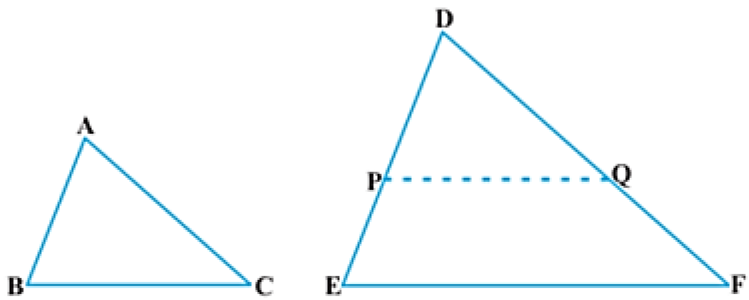
Example: Two triangles ABC and DEF exist such that $\frac{AB}{DE} = \frac{AC}{DF} < 1$ and $\angle A = \angle D$, prove that the triangles are similar.

To Prove: $\triangle ABC \sim \Delta DEF$

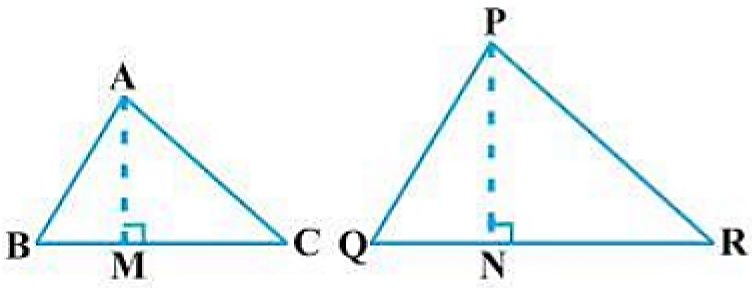
Proof: For the triangle DEF, we mark point P such that DP = AB and mark point Q such that $DQ = AC$. And join PQ.

Now, $PQ \parallel EF$ and $\triangle ABC \cong \triangle DPQ$. Hence, $\angle A = \angle D, \angle B = \angle P$, and $\angle C = \angle Q$

Therefore, $\triangle ABC \sim \Delta DEF$.



- Theorem: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.



Example: Two triangles ABC and PQR exist such that $\triangle ABC \sim \triangle PQR$.

To Prove: $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Proof: Firstly, we draw altitudes AM and PN of the triangles.

$\Rightarrow$ So, $ar(ABC) = \frac{1}{2} \times BC \times AM$ and $ar(PQR) = \frac{1}{2} \times QR \times PN$

$\Rightarrow$ So $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM)}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN}$

$\Rightarrow$ Given, $\triangle ABC \sim \triangle PQR,\ so,\ \angle B = \angle Q$

$\Rightarrow \angle M = \angle N$ (As the measure of both angles is 90°)

$\Rightarrow$ So, $\Delta ABM \sim \Delta PQN$ (As per AA similarity criterion).

Thus, $\frac{AM}{PN} = \frac{AB}{PQ}$

Given, $\triangle ABC \sim \Delta PQR,\ so \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ .... (3)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ} \times \frac{AM}{PN}\right)$ (From 1 and 3)

$\Rightarrow \left(\frac{AB}{PQ} \times \frac{AB}{PQ}\right)$ $(from\ 2)$

$\Rightarrow = (AB/PQ)^2$

Now, from (3), we get,

$\Rightarrow \frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

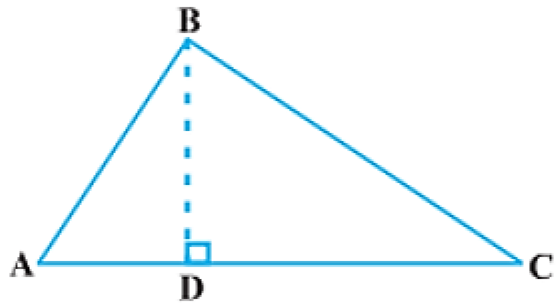
- Theorem: If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

- Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

  Given: A right triangle ABC right angled at B.

  To Prove: $AC^2 = AB^2 + BC^2$

  Proof: We draw BD ⊥ AC.



As per the previous theorem, we can write, $\Delta ADB \sim \Delta ABC$.

So, $\frac{AD}{AB} = \frac{AB}{AC}$ =(Since sides are proportional) i.e. $AD \times AC = AB^2$....... (1)

Similarly, as per the previous theorem, we can write, $\Delta BDC \sim \Delta ABC$.

So, $\frac{CD}{BC} = \frac{BC}{AC}$ = (Since sides are proportional) i.e. $CD \times AC = BC^2$........ (2)

On adding (1) and (2), we get,

$\Rightarrow AD \times AC + CD \times AC = AB^2 + BC^2$

$\Rightarrow AC(AD + CD) = AB^2 + BC^2$

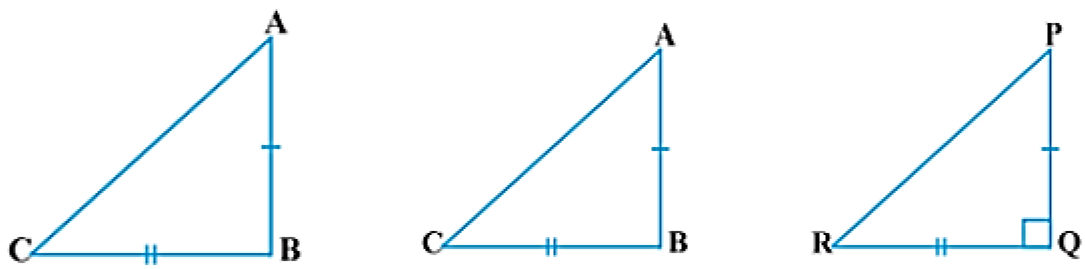
$\Rightarrow AC \times AC = AB^2 + BC^2$

$\Rightarrow AC^2 = AB^2 + BC^2$

- Theorem: In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

  Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$

  To Prove: $\angle B = 90^\circ$



Proof: Firstly, we construct a ΔPQR right-angled at Q such that $PQ = AB$ and $QR = BC$. Now, fromΔPQR, we get,

$\Rightarrow PR^2 = PQ^2 + QR^2$ (by Pythagoras theorem)
$\Rightarrow PR^2 = AB^2 + BC^2$ (since $PQ = AB$ and $QR = BC$).......... (1)
Given, $AC^2 = AB^2 + BC^2$, so, $AC = PR$........... (2)
Now, for $\triangle ABC$ and ΔPQR, $AB = PQ$, $BC = QR$, $AC = PR$.
Thus, $\Delta ABC \cong \triangle PQR$ (as per SSS congruence)
Hence, $\angle B = \angle Q$ (CPCT)

But $\angle Q = 90^\circ$ (as per construction).
$\Rightarrow So, \angle B = 90^\circ$