Board –CBSE | Class –10th | Topic – Polynomial

1. Polynomial:

The expression which contains one or more terms with non-zero coefficient is called a polynomial. A polynomial can have any number of terms.

Example: 10, $a + b$, $7x + y + 5$, $w + x + y + z$, etc. are some polynomials.

2. Degree of polynomial: The highest power of the variable in a polynomial is called the degree of the polynomial.

Example: The degree of $p(x) = x^5 - x^3 + 7$ is 5.

3. Linear polynomial:

A polynomial of degree one is called a linear polynomial.

Example: $1/(2x - 7)$, $\sqrt{s} + 5$, etc. are some linear polynomials.

4. Quadratic polynomial: A polynomial having the highest degree of two is called a quadratic polynomial. The term 'quadratic' is derived from the word 'quadrate' which means square. In general, a quadratic polynomial can be expressed in the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constants.

Example: $x^2 - 9$, $a^2 + a + 7$, etc. are some quadratic polynomials.

5. Cubic Polynomial: A polynomial having the highest degree of three is called a cubic polynomial. In general, a quadratic polynomial can be expressed in the form $ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, d are constants.

Example: $x^3 - 9x + 2$, $a^3 + a^2 + \sqrt{a} + 7$, etc. are some cubic polynomials.

6. Zeros of a Polynomial: The value of variable for which the polynomial becomes zero is called the zeros of the polynomial. In general, if k is a zero of $p(x) = ax + b$, then $p(k) = ak + b = 0$, i.e., $k = -b/a$.

Hence, zero of the linear polynomial $ax + b$ is $-b/a = -$ (Constant term)/(coefficient of x)

Example: Consider $p(x) = x + 2$. Find zeros of this polynomial.

If we put $x = -2$ in $p(x)$, we get,

$p(-2) = -2 + 2 = 0$.

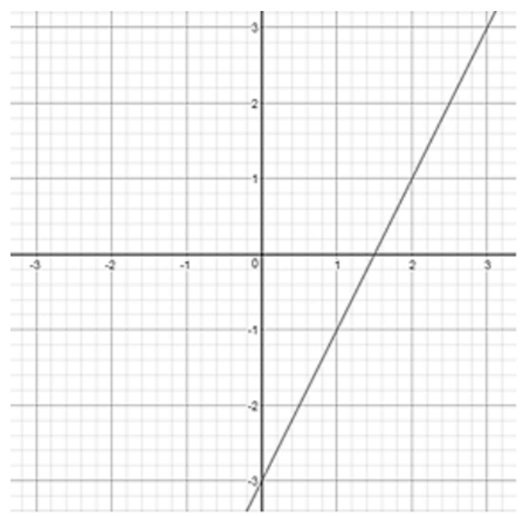
Thus, -2 is a zero of the polynomial $p(x)$

7. Geometrical Meaning of the Zeroes of a Polynomial:

(i) For Linear Polynomial:

In general, for a linear polynomial $ax + b$, $a \neq 0$, the graph of $y = ax + b$ is a straight line that intersects the x-axis at exactly one point, namely, $(-b/a, 0)$. Therefore, the linear polynomial $ax + b$, $a \neq 0$, has exactly one zero, namely, the x-coordinate of the point where the graph of $y = ax + b$ intersects the x-axis.

Example: The graph of $y = 2x - 3$ is a straight line passing through points $(0, -3)$ and $(3/2, 0)$.



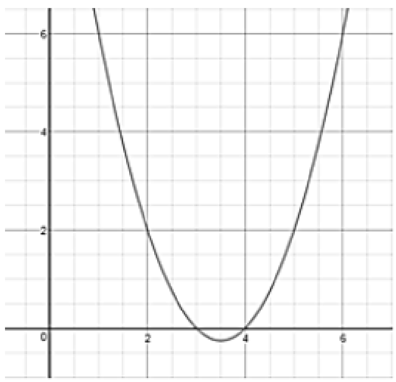
x	0	3/2
$y = 2x - 3$	6	0

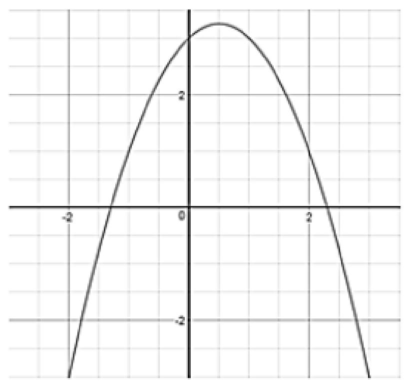
Here, the graph of $y = 2x - 3$ is a straight line that intersects the x-axis at exactly one point, namely, $(3/2, 0)$.

(ii) For Quadratic Polynomial:

In general, for any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like a curve or open downwards like curve depending on whether $a > 0$ or $a < 0$. (These curves are called parabolas.)

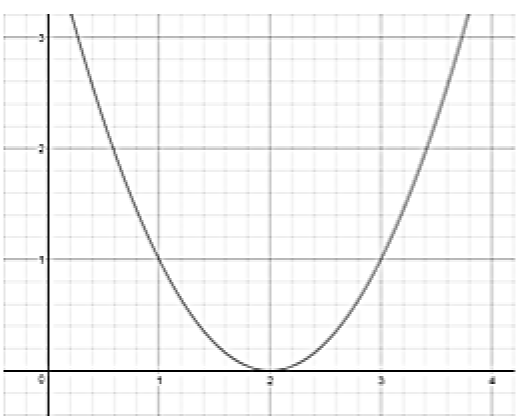
Case 1: The Graph cuts the x-axis at two distinct points.

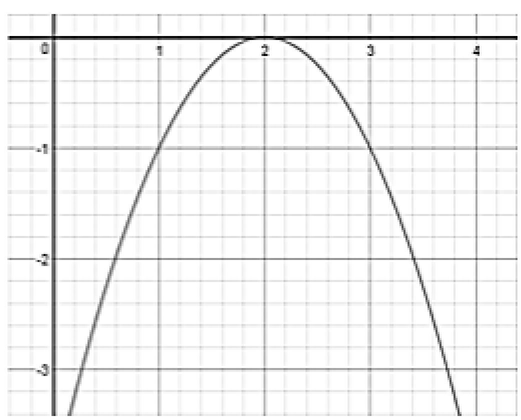




The x-coordinates of the quadratic polynomial $ax^2 + bx + c$ have two zeros in this case

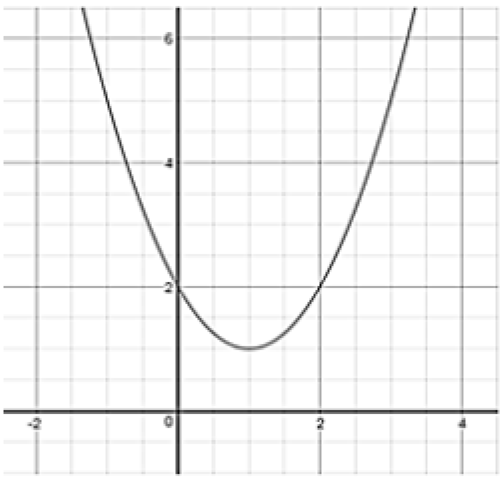
Case 2: The graph cuts the x-axis at exactly one point

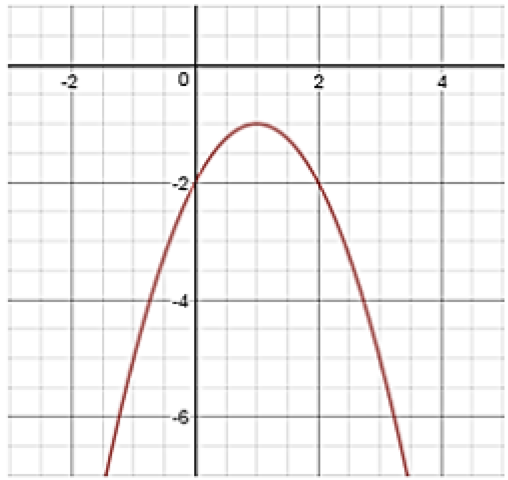




The x-coordinates of the quadratic polynomial $ax^2 + bx + c$ have only one zero in this case

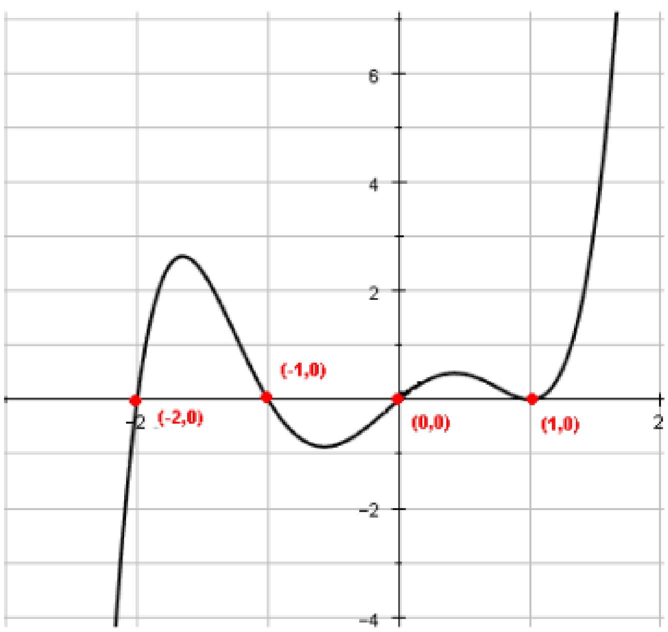
Case 3: The graph is completely above the x-axis or below the x-axis.





The quadratic polynomial $ax^2 + bx + c$ has no zero in this case

Example: For the given graph, find the number of zeroes of p(x). From the figure, we can see that the graph intersects the x-axis at four points. Therefore, the number of zeroes is 4.



8. Relationship between Zeroes and Coefficients of a Polynomial:

(i) Quadratic Polynomial:

In general, if α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then we know that $(x - \alpha)$ and $(x - \beta)$ are the factors of $p(x)$.

Moreover, $\alpha + \beta = - b/a$ and $\alpha\beta = c/a$

In general, the sum of zeros = $-$ (Coefficient of x)/(Coefficient of x^2)

Product of zeros = (Constant term)/ (Coefficient of x^2)

Example: Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zeroes and the coefficients.

On finding the factors of $x^2 + 7x + 10$, we get, $x^2 + 7x + 10 = (x + 2)(x + 5)$

Thus, the value of $x^2 + 7x + 10$ is zero for $(x + 2) = 0$ or $(x + 5) = 0$.

Or in other words, the value of $x^2 + 7x + 10$is zero for $x =- 2$ or $x =- 5$

Hence, zeros of $x^2 + 7x + 10$ are $- 2$ and $- 5$.

Now, the sum of zeros = $- 2 + (- 5) =- 7 =- 7/1$

= - (Coefficient of x)/(Coefficient of x^2)

Similarly, the product of zeros = $(- 2)\times(- 5) = 10 = 10/1$

= (Constant term) / (Coefficient of x^2)

Example: Find a quadratic polynomial for the given numbers as the sum and product of its zeroes respectively: 4,1.

Let the quadratic polynomial be $ax^2 + bx + c$.

Given, $\alpha + \beta = 4 = 4/1 = - b/a$.

$\alpha\beta = 1 = 1/1 = c/a$

Thus, $a = 1$, $b = - 4$, and $c = 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

(ii) Cubic Polynomial: In general, it can be proved that if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then,

The sum of roots $\alpha + \beta + \gamma = - b/a$

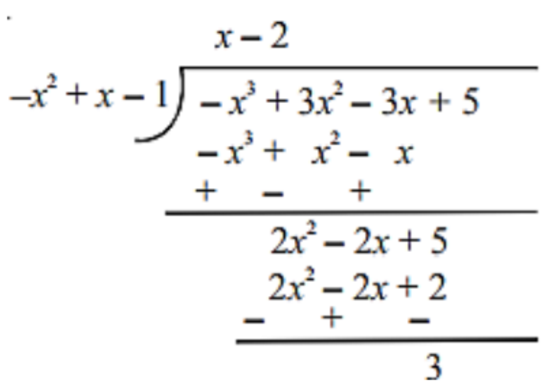
The sum of the product of two roots $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

Product of roots $\alpha\beta\gamma = \ - d/a$

9. Division Algorithm for Polynomials : If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Example: Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

On dividing $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, we get,



Here, the quotient is $(x - 2)$ and the remainder is 3.

Now, as per the division algorithm, Divisor x Quotient + Remainder = Dividend

$LHS = \left(- x^2 + x + 1\right)(x - 2) + 3$

$= \left(- x^3 + x^2 - x + 2x^2 - 2x + 2 + 3\right)$

$= \left(- x^3 + 3x^2 - 3x + 5\right)$

$RHS = \left(- x^3 + 3x^2 - 3x + 5\right)$

Thus, the division algorithm is verified.

Example: On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $(x - 2)$ and $(-2x + 4)$, respectively. Find $g(x)$.

Given, dividend $= p(x) = \left(x^3 - 3x^2 + x + 2\right)$, quotient $= (x - 2)$, remainder $= (-2x + 4)$.

Let the divisor be denoted by $g(x)$.

Now, as per the division algorithm,

Divisor × Quotient + Remainder = Dividend

$\left(x^3 - 3x^2 + x + 2\right) = g(x)(x - 2) + (-2x + 4)$

$\left(x^3 - 3x^2 + x + 2 + 2x - 4\right) = g(x)(x - 2)\left(x^3 - 3x^2 + 3x - 2\right) = g(x)(x - 2)$

Hence, $g(x)$ is the quotient when we divide $\left(x^3 - 3x^2 + 3x - 2\right)$ by $(x - 2)$.

Therefore, $g(x) = \left(x^2 - x + 1\right)$