

Board – NCERT

Class – 10th

Topic – Arithmetic Progression 5.1

Q.1 In which of the following situations, does the list of numbers involved make an arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Sol: (i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km .

It can be observed that

$$\text{Taxi fare for } 1^{\text{st}} \text{ km} = 15$$

$$\text{Taxi fare for first } 2 \text{ km} = 15 + 8 = 23$$

$$\text{Taxi fare for first } 3 \text{ km} = 23 + 8 = 31$$

$$\text{Taxi fare for first } 4 \text{ km} = 31 + 8 = 39$$

Clearly 15, 23, 31, 39 ... forms an A.P. because every term is 8 more than the preceding term.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Let the initial volume of air in a cylinder be V lit.

In each stroke, the vacuum pump removes the air remaining in the cylinder at a time.

In other words, after every stroke, only $1 - \frac{1}{4} = \frac{3}{4}$ th part of air will remain.

Therefore, volumes will be $V, \left(\frac{3V}{4}\right), \left(\frac{3V}{4}\right)^2, \left(\frac{3V}{4}\right)^3 \dots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

Cost of digging for first meter = 150

Cost of digging for first 2 meters = $150 + 50 = 200$

Cost of digging for first 3 meters = $200 + 50 = 250$

Cost of digging for first 4 meters = $250 + 50 = 300$

Clearly, 150, 200, 250, 300... forms an A.P. because every term is 50 more than the preceding term.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

We know that if Rs P is deposited at $r\%$ compound interest per annum for n years, the money will be $P\left(1 + \frac{r}{100}\right)^n$ after n years.

Therefore, after every year, the money will be

$10000\left(1 + \frac{8}{100}\right), 10000\left(1 + \frac{8}{100}\right)^2, 10000\left(1 + \frac{8}{100}\right)^3, 10000\left(1 + \frac{8}{100}\right)^4, \dots$

Clearly, adjacent terms of this series do not have the same difference between them.

Therefore, this is not an A.P.

Q.2 Write first four terms of the A.P. when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Sol: (i) $a = 10, d = 10$

$$a = 10, d = 10$$

Let the series be a_1, a_2, a_3, a_4 , as ...

$$a_1 = a = 10$$

$$a_2 = a_1 + d = 10 + 10 = 20$$

$$a_3 = a_2 + d = 20 + 10 = 30$$

$$a_4 = a_3 + d = 30 + 10 = 40$$

$$a_5 = a_4 + d = 40 + 10 = 50$$

Therefore, the series will be 10, 20, 30, 40, 50...

The first four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2, d = 0$

$$a = -2, d = 0$$

Let the series be a_1, a_2, a_3, a_4, a_5 ...

$$a_1 = a = -2$$

$$a_2 = a_1 + d = -2 + 0 = -2$$

$$a_3 = a_2 + d = -2 + 0 = -2$$

$$a_4 = a_3 + d = -2 + 0 = -2$$

Therefore, the series will be $-2, -2, -2, -2$...

The first four terms of this A.P. will be $-2, -2, -2$, and -2 .

(iii) $a = 4, d = -3$

$$a = 4, d = -3$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = 4$$

$$a_2 = a_1 + d = 4 - 3 = 1$$

$$a_3 = a_2 + d = 1 - 3 = -2$$

$$a_4 = a_3 + d = -2 - 3 = -5$$

Therefore, the series will be 4, 1, -2, -5...

The first four terms of this A.P. will be 4, 1, -2, and -5.

(iv) $a = -1, d = 1/2$

$$a = -1, d = 1/2$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = -1$$

$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Therefore, the series will be $-1, -\frac{1}{2}, 0, \frac{1}{2} \dots$

The first four terms of this A.P. will be $-1, -\frac{1}{2}, 0, \frac{1}{2} \dots$

(v) $a = -1.25, d = -0.25$

$$a = -1.25, d = -0.25$$

Let the series be $a_1, a_2, a_3, a_4 \dots$

$$a_1 = a = -1.25$$

$$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$$

$$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$$

$$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$$

Therefore, the series will be 1.25, -1.50, -1.75, -2.00.....

The first four terms of this A.P. will be -1.25, -1.50, -1.75, and -2.00.

Q.3 For the following A.P.s, write the first term and the common difference.

(i) 3, 1, -1, -3...

(ii) 5, -1, 3, 7...

(iii) 0.6, 1.7, 2.8, 3.9...

(iv) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \dots$

Sol: (i) 3, 1, -1, -3...

Here, first term, $a = 3$

The common difference, $d = \text{Second term} - \text{First term}$

$$d = 1 - 3 = -2$$

(ii) 5, -1, 3, 7...

Here, first term, $a = -5$

Common difference, $d = \text{Second term} - \text{First term}$

$$= (-1) - (-5) = -1 + 5 = 4$$

(iii) 0.6, 1.7, 2.8, 3.9...

Here, first term, $a = 0.6$

The common difference, $d = \text{Second term} - \text{First term}$

$$d = 1.7 - 0.6 = 1.1$$

(iv) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \dots$

Here, first term, $a = \frac{1}{3}$

The common difference, $d = \text{Second term} - \text{First term}$

$$d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$