Board – NCRT | Class – 10th | Topic – Arithmetic Progression 5.2

Q.1 Fill in the blanks in the following table, given that a is the first term, d the common difference, and the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	.....
II	-18	.....	10	0
III	.....	-3	18	-5
IV	-18.9	2.5	.....	3.6
v	3.5	0	105	.....

Sol: $I.\ a = 7,\ d = 3,\ n = 8,\ a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1)d$

$= 7 + (8 - 1)3$

$= 7 + (7)3 = 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that

$a = -18,\ n = 10,\ a_n = 0,\ d = ?$

We know that,

$a_n = a + (n - 1)d$

$\Rightarrow 0 = -18 + (10 - 1)d$

$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

III. Given that

$d = -3,\ n = 18,\ a_n = -5$

We know that,

$a_n = a + (n - 1)d - 5$

$= a + (18 - 1)(-3) - 5$

$= a + (17)(-3) - 5$

$= a - 51a = 51 - 5 = 46$

Hence, $a = 46$

$IV.\ a = -18.9,\ d = 2.5,\ a_n = 3.6,\ n = ?$

We know that,

$a_n = a + (n - 1)d$

$3.6 = -18.9 + (n - 1)2.53.6 + 18.9$

$= (n - 1)2.522.5 = (n - 1)2.5(n - 1)$

$= \frac{22.5}{2.5} n - 1 = 9$

$\Rightarrow n = 10$

Hence, $n = 10$

$V.\ a = 3.5,\ d = 0,\ n = 105,\ a_n = ?$

We know that,

$a_n = a + (n - 1)d$

$a_n = 3.5 + (105 - 1)0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Q.2 Choose the correct choice in the following and justify

I. 30^{th} term of the A.P: 10, 7, 4, ..., is

A. 97

B. 77

C. -77

D. -87

II. 11^{th} term of the A.P: $-3, \frac{-1}{2}, 2, \ldots$ is

A. 28 B. 22 C. -38 D. $-48\frac{1}{2}$

Sol: I. 30^{th} term of the A.P: 10, 7, 4, ..., is

Given that

A.P. 10, 7, 4, ...

First-term, $a = 10$

The common difference, $d = a_2 - a_1 = 7 - 10 = -3$

We know that, an $= a + (n - 1)d$

$a_{30} = 10 + (30 - 1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence, $a_{30} = -77$

II. 11^{th} term of the A.P: $-3, \frac{-1}{2}, 2, \ldots$ is

Given that A.P. $-3, \frac{-1}{2}, 2, \ldots$

First-term, $a = -3$

The common difference, $d = a_2 - a_1 = 7 - 10 = -3$

$= \frac{1}{2} - (-3) = \frac{1}{2} + 3 = \frac{5}{2}$

we know that,

$a_n = a + (n - 1)d$

$a_{11} = -3 + (11 - 1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 10\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = 22$

Hence, the a_{11} is 22.

Q.3 In the following APs find the missing term in the box.

I. 2, □, 6

II. □, 13, □, 3

III 5, □, □, $9\frac{1}{2}$

iv. -4, □, □, □, □, 6

v. □, 38, □ □, □, -22

sol: **I. 2,** □, 6

For this A.P., $a = 2$ and $a_3 = 26$

We know that, $an = a + (n - 1)d$

$\Rightarrow a_3 = 2 + (3 - 1)d$

$\Rightarrow 26 = 2 + 2d$

$\Rightarrow 24 = 2d$

$\Rightarrow d = 12$

$a_2 = 2 + (2 - 1)12 = 14$

Hence, $a_2 = 14$

II. □, 13, □, 3

For this A.P., $a_2 = 13$ and $a_4 = 3$

We know that, an = $a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$13 = a + d$ ..... (i)

$a_4 = a + (4 - 1)d$

$3 = a + 3d$ ..... (ii)

On subtracting (I) from (II), we obtain

$-10 = 2d$

$d = -5$

From equation (I), we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)(-5) = 18 + 2(-5)$

$a_3 = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

(iii) 5, □, □, $9\frac{1}{2}$

For this A.P., $a = 5$ and $a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that, $a_n = a + (n - 1)d$

$a_4 = a + (4 - 1)d$

$\frac{19}{2} = 5 + 3d$

$\Rightarrow \frac{19}{2} - 5 = 3d$

$\Rightarrow \frac{9}{2} = 3d$

$\Rightarrow d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are $\frac{13}{2}$, and 8 respectively.

(iv) -4, □, □, □, □, 6

For this A.P.,

For this A.P., $a = -4$ and $a6 = 6$

We know that,

$an = a + (n - 1)d$

$a_6 = a + (6 - 1)d$

$6 = -4 + 5d$

$10 = 5d \Rightarrow d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are -2, 0, 2, and 4 respectively.

(v) □, 38, □ □, □, -22

For this A.P., $a_2 = 38$ and $a_6 = -22$

We know that

$a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$38 = a + d$ ..... (1)

$a_6 = a + (6 - 1)d$

$-22 = a + 5d$ .... (2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15 a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are 53, 23, 8, and -7 respectively.

Q.4 Which term of the A.P. 3, 8, 13, 18, … is 78?

Sol: 3, 8, 13, 18, …

For this A.P., $a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$75 = (n - 1)5$

$(n - 1) = 15$

$\Rightarrow n = 16$

Hence, 16^{th} term of this A.P. is 78.

Q.5 Find the number of terms in the following A.P.

$I.$ 7, 13, 19, ..., 205

II. 18, $15\frac{1}{2}$, 13, ..., -47

Sol: **I.** 7, 13, 19, ..., 205

7, 13, 19, ..., 205

For this A.P., $a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this A.P. $a_n = 205$

We know that,

$a_n = a + (n - 1)d$

Therefore, $205 = 7 + (n - 1)6$

$198 = (n - 1)6$

$33 = (n - 1)$

$n = 34$

Therefore, this given series has 34 terms in it.

$II.$ 18, $15\frac{1}{2}$, 13, ..., -47

18, $15\frac{1}{2}$, 13, ..., -47

For this A.P., a = 18

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31-36}{2} = \frac{-5}{2}$

Let there be n term in this A.P.

Therefore, $a_n = -47$ and we know that,

$a_n = a + (n-1)d$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$

$-65 = (n-1)\left(-\frac{5}{2}\right)$

$(n-1) = \frac{-130}{-5} = 26$

$n = 27$

Therefore, the given A.P. has 27 terms in it.

Q.6 Check whether -150 is a term of the A.P. 11, 8, 5, 2, …

Sol: For this A.P., $a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let there are n terms in this A.P.

We know that

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$-164 = -3n$

$n = 164/3$

Clearly, n is not an integer.

Therefore, -150 is not a term of this A.P.

Q.7 Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73

Sol: $a_{11} = 38$ and $a_{16} = 73$

We know that,

$a_n = a + (n - 1)d$

$a_{11} = a + (11 - 1)d$

$38 = a + 10d$ .... (1)

Similarly,

$a_{16} = a + (16 - 1)d$

$73 = a + 15d$ ..... (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10\times(7)$

$38 - 70 = a$

$a = - 32$

$a_{31} = a + (31 - 1)d = - 32 + 30(7)$

$a_{31} = - 32 + 210 = 178$

Hence, 31^{st} term is 178 .

Q.8 An A.P. consists of 50 terms of which 3 rd term is 12 and the last term is 106 . Find the 29^{th} term.

Sol: Given that,

$a_3 = 12$ and $a_{50} = 106$

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$12 = a + 2d$ ..... (I)

Similarly, $a_{50} = a + (50 - 1)d$

$106 = a + 49d$ ..... (II)

On subtracting (I) from (II), we obtain

$94 = 47d$

$d = 2$

From equation (I), we obtain

$12 = a + 2(2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore, 29^{th} term is 64.

Q.9 If the $3rd$ and the 9 th terms of an A.P. are 4 and $-\ 8$ respectively. Which term of this A.P. is 0 .

Sol: Given that,

$a_3 = 4$ and $a_9 =- 8$

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$4 = a + 2d$

$a_9 = a + (9 - 1)d$

$-8 = a + 8d$

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$an = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Q.10 If 17^{th} term of an A.P. exceeds its 10^{th} term by 7 . Find the common difference.

Sol: For an A.P., an = $a + (n - 1)d$

$a_{17} = a + (17 - 1)d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$\Rightarrow d = 1$

Therefore, the common difference is 1.

Q.11 Which term of the A.P. $3, 15, 27, 39, \ldots$ will be 132 more than its 54^{th} term?

Sol: Given A.P. is 3, 15, 27, 39, ...

$a = 3d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)d = 3 + (53)(12)$

$a_{54} = 3 + 636 = 639$

$132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1)d$

$771 = 3 + (n - 1)12$

$768 = (n - 1)12$

$12(n - 1) = 64$

$\Rightarrow n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

Q.12 Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Sol: Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1)d = a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)d$

$a_{1000} = a_1 + 999\,d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)d$

$a_{100} = a_2 + 99d$

$a_{1000} = a2 + (1000 - 1)d$

$a_{1000} = a_2 + 999\,d$

Given that, the difference between

100^{th} term of these A.P.s $= 100$

Therefore, $\left(a_1 + 99d\right) - \left(a_2 + 99d\right) = 100$

$a_1 - a_2 = 100$

Difference between 1000^{th} terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000 th terms of these A.P. will be 100.

Q.13 How many three digit numbers are divisible by 7

Sol: First three-digit number that is divisible by 7 $= 105$

Next number $= 105 + 7 = 112$

Therefore, 105, 112, 119, ...

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999 . When we divide it by 7 , the remainder will be 5 .

Clearly, $999 - 5 = 994$ is the maximum possible three-digit number that is divisible by 7

The series is as follows.

105, 112, 119, ..., 994

Let 994 be the n^{th} term of this A.P.

$a = 105,\ d = 7$ and $a_n = 994$

$n =?\ an = a + (n - 1)d994 = 105 + (n - 1)7889 = (n - 1)7(n - 1) = 127 \Rightarrow n = 128$

Therefore, 128 three-digit numbers are divisible by 7 .

Q.14 How many multiples of 4 lie between 10 and 250?

Sol: First multiple of 4 that is greater than 10 is 12 . Next will be 16 .

Therefore, 12, 16, 20, 24, ...

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2 .

Therefore, $250 - 2 = 248$ is divisible by 4

12, 16, 20, 24, ..., 248

Let 248 be the n

th term of this A.P. $a = 12,\ d = 4$ and $a_n = 248$

$an = a + (n - 1)d248 = 12 + (n - 1)4236/4 = (n - 1)(n - 1) = 59n = 60$

Therefore, there are 60 multiples of 4 between 10 and 250 .

Q.15 For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, ... equal

Sol: 63, 65, 67, ...

$a = 63\ d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this A.P. $= a_n = a + (n - 1)d$

$a_n = 63 + (n - 1)2 = 63 + 2n - 2$

$a_n = 61 + 2n$...(1)

3, 10, 17, ...

$a = 3$ and $d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. $= 3 + (n - 1)7$

$a_n = 3 + 7n - 7a_n = 7n - 4$

It is given that, n^{th} term of these A.P.s are equal to each other. Equating both these equations, we obtain

$61 + 2n = 7n - 461 + 4 = 5n5n = 65 \Rightarrow n = 13$

Therefore, 13 th terms of both these A.P.s are equal to each other.

Q.16 Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12 .

Sol: $a_3 = 16$

$a + (3 - 1)d = 16a + 2d = 16(1)a_7 - a5 = 12[a + (7 - 1)d] - [a + (5 - 1)d] = 12(a + 6d) -$

$2\,d = 12 \Rightarrow d = 6$

From equation (1), we obtain

$a + 2(6) = 16a + 12 = 16a = 4$

Therefore, A.P. will be 4, 10, 16, 22, ...

Q.17 Find the 20 th term from the last term of the A.P. 3, 8, 13, ..., 253

Sol: Given A.P. is 3, 8, 13, ..., 253

Common difference for this A.P. is 5 .

Therefore, this A.P. can be written in reverse order as

253, 248, 243, ..., 13, 8, 5

For this A.P.,

$a = 253\, d = 248 - 253 =- 5n = 20a20 = a + (20 - 1)da20 = 253 + (19)(- 5)a20 = 253 - 95$

Therefore, 20 th term from the last term is 158 .

Q.18 The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 44 .

Find the first three terms of the A.P.

Sol: We know that,

$an = a + (n - 1)da_4 = a + (4 - 1)d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7da_6 = a + 5da_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 242a + 10d = 24a + 5d = 12a_6 + a_{10} = 44a + 5d + a + 9d = 442a + 14d$

On subtracting equation (1) from (2), we obtain

$2\,d = 22 - 12$

$2d = 10 \Rightarrow d = 5$

From equation (1), we obtain

$a + 5d = 12a + 5(5) = 12a + 25 = 12a =- 13a_2 = a + d =- 13 + 5 =- 8a_3 = a_2 + d =- 8 +$

Therefore, the first three terms of this A.P. are $-13, -8$, and -3.

Q.19 Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Sol: It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his

salary is increased by Rs 200 .

Therefore, the salaries of each year after 1995 are 5000, 5200, 5400, …

Here, $a = 5000$ and $d = 200$

Let after n^{th} year, his salary be Rs 7000 .

Therefore, an $= a + (n - 1)d$

$7000 = 5000 + (n - 1)200200(n - 1) = 2000(n - 1) = 10 \Rightarrow n = 11$

Therefore, in 11 th year, his salary will be Rs 7000 .

Q.20 Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her week, her weekly savings become Rs 20. 75, find n.

Sol: Given that,

$a = 5$ and $d = 1.75$

$an = 20.75, n = ? 20.75 = 5 + (n - 1)1.75(n - 1) = \frac{15.75}{1.75} = \frac{1575}{175} = \frac{63}{7} = 9n - 1 = 9 \Rightarrow n = 10$

Hence, n is 10 .

$an = a + (n - 1)dn - 1 = 9 \Rightarrow n = 10$

Hence, n is 10.