Board – NCRT | **Class – 10th** | **Topic – Quadratic 4.3**

Q.1 Find the roots of the following quadratic equations, if they exist, by the method of completing the square: $2x^2 - 7x + 3 = 0$

Sol: $2x^2 - 7x + 3 = 0$

$\Rightarrow 2x^2 - 7x + 3$

On dividing both side of the equation by 2 , we obtain

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2} \Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$

On adding $\left(\frac{7}{2}\right)^2$ to both side of equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{2}\right)^2 = \left(\frac{7}{2}\right)^2 - \frac{3}{2} \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2} \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \Rightarrow ($

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ or $x = \frac{2}{4}$

$\Rightarrow x = 3$ or $\frac{1}{2}$

Q.2 Find the roots of the following quadratic equations, if they exist, by the method of completing the square: $2x^2 + x - 4 = 0$

Sol: $2x^2 + x - 4 = 0 \Rightarrow 2x^2 + x - 4$

On dividing both sides of the equation by 2 , we obtain

$\Rightarrow x^2 - \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both side of equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2 \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16} \Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4} \Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4} \Rightarrow x = \pm\frac{\sqrt{}}{}$

$\Rightarrow x = \frac{\sqrt{33}-1}{4}$ or $x = \frac{-\sqrt{33}-1}{4}$

Q.3 Find the roots of the following quadratic equations, if they exist, by the method of completing the square: $4x^2 + 4\sqrt{3}x + 3 = 0$

Sol: $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2\times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0 \Rightarrow (2x + \sqrt{3})^2 = 0$

$\Rightarrow (2x + \sqrt{3}) = 0$ and $\Rightarrow (2x + \sqrt{3}) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

Q.4 Find the roots of the following quadratic equations, if they exist, by the method of completing the square: $2x^2 + x + 4 = 0$

Sol: $2x^2 + x + 4 = 0 \Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2 \Rightarrow (x)^2 + 2\times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both side of equation, we obtain

$\Rightarrow (x)^2 + 2\times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2 \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2 \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{31}{16} \Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4} \Rightarrow x =$

However, the square of a number cannot be negative.

Therefore, there is no root for the given equation.

Q.5 Find the roots of the quadratic equations given in Q.above by applying the quadratic formula. $2x^2\ 7x + 3 = 0$

Sol: $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{7\pm\sqrt{49-24}}{4} \Rightarrow x = \frac{7\pm\sqrt{25}}{4} \Rightarrow x = \frac{7\pm5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } x = \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$$

$$\Rightarrow x = 3 \text{ or } x = \frac{1}{2}$$

Q.6 Find the roots of the quadratic equations given in Question .above by applying the quadratic formula.

$2x^2 + x - 4 = 0$

Sol: $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2,\ b = 1, c =- 4$

By using quadratic formula, we obtain

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{-1\pm\sqrt{1+32}}{4} \Rightarrow x = \frac{-1\pm\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{-1+\sqrt{33}}{4} \text{ or } x = \frac{-1-\sqrt{33}}{4}$$

Q.7 Find the roots of the quadratic equations given in Question .above by applying the quadratic formula.

$4x^2 + 4\sqrt{3}x + 3 = 0$

Sol: $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 4, b = 4\sqrt{3}, c = 3$ By using quadratic formula, we obtain

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{-4\sqrt{3}\pm\sqrt{48-48}}{8} \Rightarrow x = \frac{-4\sqrt{3}\pm0}{8}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \text{ or } x = \frac{-\sqrt{3}}{2}$$

Q.8 Find the roots of the quadratic equations given in Question .above by applying the quadratic formula. $2x^2 + x + 4 = 0$

Sol: $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 4,\ b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{-1\pm\sqrt{1-32}}{4} \Rightarrow x = \frac{-1\pm\sqrt{-31}}{4}$

However, the square of a number cannot be negative.

Therefore, there is no root for the given equation.

Q.9 Find the roots of the following equations by factorization: $x - \frac{1}{x} = 3, x\neq 0$

Sol: $x - \frac{1}{x} = 3, x\neq 0 \Rightarrow x^2 - 3x - 1 = 0$

on com min g this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b =- 3, c =- 1$

By usin g quadratic formula, we obtain

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{3\pm\sqrt{9+4}}{2} \Rightarrow x = \frac{3\pm\sqrt{13}}{2}$

$\therefore x = \frac{3+\sqrt{13}}{2}$ or $x = \frac{3-\sqrt{13}}{2}$

Q.10 Find the roots of the following quadratic $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x\neq - 4, 7$

Sol: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\Rightarrow \frac{x-7-x-4}{(x+4)(x+7)} = \frac{11}{30} \Rightarrow (x + 4)(x + 7) = 30 \Rightarrow x^2 - 3x - 28 =- 30 \Rightarrow x^2 - 2x - x + 2 = 0 \Rightarrow (x - 2)$

$\Rightarrow x = 1$ or 2

Q.11 The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol: Let the present age of Rehman be x years. Three years ago, his age was $(x - 3)$ years.

Five years hence, his age will be $(x + 5)$ years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \Rightarrow \frac{x+5-x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow 2(2x + 2) = (x - 3)(x + 5) \Rightarrow 6x + 6 = x^2 + 2$

$\Rightarrow x = 7 \text{ or} - 3$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Q.12 In a class test, the sum of Shefali's marks in Mathematics and English is 30 . Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210 . Find her marks in the two subjects.

Sol: Let the marks in Math's be x.

Then, the marks in English will be $30 - x$.

According to the given question,

$(x + 2)(30 - x - 3) = 210(x + 2)(27 - x) = 210 \Rightarrow - x^2 + 25x + 54 = 210 \Rightarrow x^2 - 25x + 156 =$

If the marks in Maths are 12 , then marks in English will be $30 - 12 = 18$

If the marks in Maths are 13 , then marks in English will be $30 - 13 = 17$

Q.13 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol: Let the shorter side of the rectangle be xm.

Then, larger side of the rectangle $= (x + 30)m$

Diagonal of the triangle $= \sqrt{x^2 + (x + 30)^2}m$

itis given that the diagonal of the rec tan gle is $60mmore$ than the shorter side,

$\therefore \sqrt{x^2 + (x + 30)^2} = x + 60 \Rightarrow x^2 + (x + 30)^2 = (x + 60)^2 \Rightarrow 6x + 6 = x^2 + 2x - 15 \Rightarrow x^2 - 60x -$

$\Rightarrow x = 90 \text{ or} - 30$

However, side cannot be negative. Therefore, the length of the shorter side will be $90\ m$.

Hence, length of the larger side will be $(90 + 30)m = 120\ m$

Q.14 The difference of squares of two numbers is 180 . The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol: Let the larger and smaller number be x and y respectively.

According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 - 8x = 180 \Rightarrow x^2 - 8x - 180 = 0 \Rightarrow x^2 - 18x + 10x - 180 = 0 \Rightarrow (x - 18)(x + 10) = 0 \Rightarrow x$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$x = 18 \therefore y^2 = 8x = 8\times18 = 144 \Rightarrow y = \pm\sqrt{144} = \pm12$

$\therefore$ smaller number $= \pm12$

Therefore, the numbers are 18 and 12 or 18 and $-\ 12$.

Q.15 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol: Let the speed of the train be $\frac{xkm}{hr}$. Time taken to cover 360 $kmhr$

According to the given question,

$(x + 5)\left(\frac{360}{x} - 1\right) = 360 \Rightarrow (x + 5)\left(\frac{360}{x} - 1\right) = 360 \Rightarrow 360 - x + \frac{1800}{x} - 5 = 360 \Rightarrow x^2 + 5x - 1800$

However, speed cannot be negative.

Therefore, the speed of train is $\frac{40\ km}{h}$

Q.16 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol: Let the time taken by the smaller pipe to fill the tank be xhr.

Time taken by the larger pipe $= (x - 10)hr$

Part of tank filled by smaller pipe in 1 hour $= \frac{1}{x}$

Part of tank filled by larger pipe in 1 hour $= \frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{8}{75}$ hours by both the pipes together.

Therefore,

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}\frac{x-10+x}{x(x-10)} = \frac{8}{75} \Rightarrow 75(2x - 10) = 8x^2 - 80x \Rightarrow 150x - 750 = 8x^2 - 80x \Rightarrow 8x^2 - 230x +$

$i.e.,\ = 25, \frac{80}{3}$

Time taken by the smaller pipe cannot be $= 3.75$ hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible. Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and $25 - 10 = 15$ hours respectively.

Q.17 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Sol: Let the average speed of passenger train be xkm/h.

Average speed of express train $= (x + 11)km/h$

It is given that the time taken by the express train to cover 132 km is

1 hour less than the passenger train to cover the same distance.

$\frac{132}{x} - \frac{132}{x+11} = 1\frac{132}{x} - \frac{132}{x+11} = 1 \Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1 \Rightarrow \frac{132\times11}{x(x+11)} = 1 \Rightarrow 132\times11 = x(x + 11) \Rightarrow x^2 + 11x$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be $33 + 11 = 44\ km/h$.

Q.18 Sum of the areas of two squares is $468\ m^2$. If the difference of their perimeters is $24\ m$, find the sides of the two squares.

Sol: Let the sides of the two squares be xm and ym. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively. It is given that

$4x - 4y = 24x - y = 6x = y + 6$

$\Rightarrow x^2 + y^2 = 468 \Rightarrow (6 + y)^2 + y^2 = 468 \Rightarrow 36 + y^2 + 12y + y^2 = 468 \Rightarrow 2y^2 + 12y - 432 = 0 \Rightarrow y^2 +$

$y = -18$ or 12

However, side of a square cannot be negative.

Hence, the sides of the squares are $12\ m$ and $(12 + 6)m = 18\ m$