

Board – NCERT

Class – 10th

Topic – Real Number 1.3

Q.1 Prove that $\sqrt{5}$ is irrational.

Sol: Let $\sqrt{5}$ be a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$.

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer.

$$(5k)^2 = 5b^2$$

$$b^2 = 5b^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that is irrational.

Q.2 Prove that $3 + 2\sqrt{5}$ is irrational.

Sol: Let $3 + 2\sqrt{5}$ be a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false. Therefore, $3 + 2\sqrt{5}$ is irrational.

Q.3 Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Sol: (i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\frac{1}{\sqrt{2}} = \frac{a}{b}$, $\sqrt{2} = \frac{b}{a}$

$\frac{b}{a}$ is a rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is a rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

since a and b are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.